NUM. I.

Determinatio Orbitæ Cometæ qui menſe Martio hujus anni 1742. potiſſimum fuit obſervatus.

AUCTORE

L. Euler.

§. I.

Quanquam plures habentur methodi paſſim traditæ ex aliquot Cometæ obſervationibus ejus orbitam determinandi, tamen non dubito, quin plerique, qui hoc tempore in Theoriam Cometæ nuper obſervati inquiſiverint, methodum cum faciliorem tum etiam certiorem deſideraverint, quam ſequerentur. Quantum enim mihi quidem conſtat, complures eximii Aſtronomi vel fruſtra in hoc negotio operam collocaverunt, vel computi moleſtiis de- *Tab. I.*

territi nequidem elaboraverunt: ac de me quidem ipſe diſſimulare non poſſum, me non parum temporis tam proprio ſtudio, quam amicorum adhortationibus permotum in hac inveſtigatione conſumſiſſe, neque tamen per methodos cognitas quicquam impetrare potuiſſe, in quo acquieſcerem. Videtur autem iſte cometa præ reliquis, qui calculo adhuc ſunt ſubjecti tot tantisque anomaliis implicatus, ut ejus motus verus, qualis ex ſole conſpici debuiſſet, ne minimum quidem conſentiret cum eo, quem hic ex terra ſpectavimus. Quare cum orbita cometæ ex ejus locis heliocentricis determinari debeat, tantum diſcrimen, quod inter loca heliocentrica & obſervata intercedit, maxime impedit, quo minus ad ipſius motus veri cognitionem pervenire queamus. Poſtquam enim obſervationes cum Theoria Newtoni contuliſſem, facile perſpexi motum cometæ, qui ex terra directus ſecundum ſignorum ſeriem & celerrimus fuit viſus, revera contra ſignorum ordinem retrogradum adeoque tardum fuiſſe, ut ejus celeritas non multum ſuperaret eam, qua terra in orbita ſua progreditur. Hinc autem ſumma naſcitur difficultas ex locis geocentricis ejus loca vera heliocentrica colligendi, quæ propterea etiam plurimum augetur, quod minimus error in aſſignatione locorum heliocentricorum commiſſus, maximam aberrationem in orbitæ determinatione producat. Quamobrem etiamſi ejus parallaxis obſervari potuiſſet, tamen niſi eſſet exactiſſima atque ad minuta ſecunda abſoluta, vix inde ullum ſubſidium ad orbitæ determinationem redundaret.

§. II.

Summus conſenſus motuum cœleſtium cum Theoria Neutoniana, qui continuo magis magisque elucet, omnino dubitare

tare nos non ſinit, quin cometæ cuncti pariter ac planetæ, quantumvis etiam ipſorum motus obſervati adverſari videantur, in ſectionibus conicis, in quarum alterutro ſoco Sol verſetur, moveantur atque æqualibus temporibus æquales areas circum ſolem conficiant. Quodſi autem motus cometæ cum motu terræ aliusve planetæ primarii circa ſolem gyrantis comparetur, erunt tempora, quibus cometa & planeta data in orbitis ſuis ſpatia abſolvunt, in ratione compoſita ex ratione arearum circa ſolem deſcriptarum directa, ac ratione laterum rectorum ſeu parametrorum reciproca ſubduplicata. Dubium itaque eſt nullum, quin cometæ hac lege in ſectionibus conicis circa ſolem revolvantur: ipſorum vero motus tantopere inæquabilis, quo modo ad ſolem ſatis prope accedunt, modo longiſſime ab eo recedunt, manifeſto indicat eorum orbitas eſſe neque circulares, neque ellipſes orbium planetarum ſimiles. Erunt itaque vel ellipſes maxime oblongæ, vel parabolæ vel adeo hyperbolæ; priori autem caſu ob orbitam in ſe redeuntem tandem redire, ſtatisque temporibus reverti deberent, reliquis vero caſibus nulla reverſio foret expectanda, niſi forte ipſorum curſus a ſtellis fixis ita immutaretur, ut ſemita ſolem verſus inflecteretur. Cum autem non ſolum veriſimile ſit cometas ſtata reverſionis tempora habere, ſed etiam ellipſis maxime oblonga a parabola ſenſibiliter non discrepet, a veritate non ſenſibiliter aberrabimus, ſi ellipſin maxime excentricam pro orbita cometæ aſſumamus; vel cum ejusmodi ellipſis cum parabola facile confundatur, ſatis tuto orbitæ cometarum inter parabolas quæri ſolent. Tam brevi enim temporis ſpatio, quo cometæ apparent, ſive in ellipſibus maxime

 oblon-

oblongis, ſive in parabolis, ſive in hyperbolis non nimis divergentibus moveantur, motus fere idem deprehendatur neceſſe eſt. Interim tamen ſi motus cometæ deinceps a parabola aliquantum deflectere obſervetur, ex hac ipſa deflexione vera orbita cometæ, utrum ellipſis ſit an hyperbola, concludi poterit; ſicque ex hypotheſi licet falſa aſſumta, tamen veritas elicietur.

§. III.

Si igitur aſſumamus orbitam cometæ eſſe parabolam, in cujus foco ſol exiſtat, quinque res occurrunt determinandæ, ut locum cometæ geocentricum ad quodvis tempus aſſignare queamus. Primo enim ſi ipſam cometæ orbitam in ſe conſideremus, ejus verticem ſeu perihelium, ejusque a Sole diſtantiam noſſe oportet; quo pacto magnitudo parabolæ definitur; hincque tempus, quo cometa datum ſpatium percurrit, definiri poterit. Secundo tempus cognitum eſſe debet, quo cometa in ipſo perihelio eſt verſatus, ut hinc locus cometæ in orbita ſua ad datum tempus colligi queat. Deinde ad locum cometæ reſpectu eclipticæ exprimendum, poſitio orbitæ reſpectu eclipticæ eſt quærenda: tertio igitur locus, ubi orbita cometæ planum eclipticæ interſecat, notus eſt debet, quæ interſectio eſt linea recta per ſolem tranſiens, ac linea nodorum appellatur; duo ſcilicet orbita cometæ habet puncta in ipſo plano eclipticæ ſita ſibi ediametro oppoſita, quorum alterum, ex quo cometa in hemisphærium boreale ascendit, nodus ascendens, alterum vero, ubi cometa in regionem auſtralem demergitur, nodus descendens vocatur: requiritur idcirco locus ulterutrius nodi, qui per longitudinem ſeu diſtantiam a puncto æquinoctali verno indicatur. Quarto diſtantia nodi a perihelio, ſeu angulus quem

axis

axis orbitæ cum linea nodorum facit, inveſtigari debet; ut ſi locus cometæ a perihelio conſtet, ſimul ejus diſtantia a nodo aſſignari poſſit. Quinto denique inclinatio orbitæ cometæ ad eclipticam eſt definienda, quæ menſuratur angulo, quem hæc duo plana orbitæ cometæ & eclipticæ inter ſe conſtituunt. His igitur quinque rebus cognitis ad datum tempus locus cometæ reſpectu eclipticæ, hoc eſt ejus longitudo & latitudo poterit inveniri, ſimulque ejus vera a Sole diſtantia; ex quibus porro, ſi locus terræ in ecliptica ad hoc idem tempus computetur, ejus locus geocentricus, hoc eſt longitudo ac latitudo geocentrica reperiri poterit. Has vero easdem quinque res cognitas eſſe oportet, ſi orbita cometæ non aſſumatur parabola, ſed alia ſectio conica puta ellipſis vel hyperbola ſpecie data; hoc eſt in qua ratio inter diſtantiam perihelii a Sole & latus rectum cognita aſſumitur, ſin autem hæc ratio eſſet incognita, ſextam conditionem quinque memoratis addendam conſtitueret.

§. IV.

Sive igitur aſſumamus pro orbita cometæ parabolam ſive aliam ſectionem conicam ſpecie datam, quinque habemus res ſeu quantitates incognitas, quas ſi problema de orbitæ cometæ inventione reſolvere velimus, determinari oportet. Prima ſcilicet incognita eſt diſtantia perihelii a ſole; ſecundo quæri debet tempus, quo cometa per perihelium tranſierit; tertio longitudo nodi ſive ascendentis ſive deſcendentis; quarto diſtantia heliocentrica nodorum a perihelio; & quinto inclinatio orbitæ ad eclipticam. Sin autem pro orbita cometæ aſſumeretur ſectio conica in genere, tum ad has quinque incognitas accederet ſexta, ratio ſcilicet lateris recti ad diſtantiam perihelii a ſole.

Quodſi ergo hoc problema more in analyſi maxime conſueto reſolvere velimus, loco harum incognitarum totidem litteræ in calculum induci, ex iisque aliquot cometæ loca geocentrica ſupputari deberent, quæ tum locis obſervatis æqualia poſita æquationes ſuppeditarent, ex quibus incognitas erui liceret. Sic ex una quaque obſervatione obtinerentur binæ æquationes, altera ex longitudine altera ex latitudine, & hanc obrem tres obſervationes ad ſex incognitas determinandas ſufficere videantur; quoniam vero orbita cometæ eſt plana, duæ latitudines jam per ſe reliquas latitudines determinant: ita ut quotcunque obſervationes cum calculo comparentur, duas tantum latitudines conſideraſſe ſufficiat. Hancobrem ex tribus obſervationibus quinque ſolum incognitæ, ideoque orbita cometæ, ſi jam ſpecie fuerit data, determinari poterunt; ac, ſi ipſa ſpecies fuerit incognita, quatuor obſervationibus ad negotium abſolvendum erit opus. At vero hæc via problema reſolvendi, quæ primo intuitu ſe offert, tantis involuta eſt difficultatibus, ut per eam nunquam optatum ſcopum attingere poſſemus, tam propter ingentem incognitarum numerum, quam ob ſummam earum permixtionem mutuam, quæ ſucceſſivam incognitarum eliminationem, ſingularumque determinationem non ſolum difficillimam ſed etiam viribus humani ingenii inſuperabilem redderet. Quam obrem hac directa ſolvendi via relicta alii alias excogitaverunt methodos indirectas, per quas incognitarum numerum diminuerent, & quæſtionem vero tantum proxime reſolverent; interim tamen vel ſumma difficultas cum calculo manſit conjuncta, vel aberratio a veritate nimis magna deprehenditur. Quocirca niſi nimium errare velimus, rigorem geome-

geometricum, quantum fieri poterit, retinere oportebit, neque veram theoriam per hypotheſes fictas contaminari conveniet, quibus calculus quidem ſublevetur, veritati autem vis inferatur.

§. V.

Poſt plura autem in ſolutione hujus problematis inſtituta tentamina, inveſtigationem ita adornari debere intellexi, ut plus una incognita in calculum non ingrediatur; per incognitarum enim multitudinem ſolutionem maxime impediri animadverti. Quanquam autem hoc ipſum initio factu difficillimum videbatur, propterea quod quinque illæ incognitæ prorſus a ſe invicem non pendeant, tamen re diligentius perpenſa, atque tribus obſervationibus conjunctim in ſubſidium vocatis, inveni tandem methodum certam arbitrariis fictionibus non inquinatam cujus ope unica incognita aſſumta, omnia quæ ad perfectam orbitæ Cometæ cognitionem pertineant determinare, atque tandem hanc ipſam incognitam non adeo difficulter definire poſſem. Via quidem conſueta non licebat negotium ad reſolutionem æquationis cujusdam, qua valor incognitæ indicaretur, perducere; verum cum unica habeatur incognita, expedire viſum eſt, ex pluribus valoribus pro ea aſſumtis, eum qui veritati eſſet conſentaneus, concludere: ſimili ſcilicet ratiocinio in hac inquiſitione uti convenit, quo in regula *Falſi* dicta per aliquot poſitiones falſas vera ſolutio erui ſolet. Hoc autem pacto prorſus evitamus litteras incognitas, quibus calculus maxime perturbaretur. Neque vero hoc compendium in uſum vocari poſſet, ſi plures una incognita determinari deberent, cum ſubſtitutio valorum arbitrariorum pro ſingulis incognitis,

gnitis, niſi eſſet fere infinita, unquam ad cognitionem unius. cujusque perduceret. Hanc igitur ſolvendi methodum ſecutus primum parabolam determinabo, in qua cometæ motus cum tribus obſervationibus conſentiat; tum vero etiam ellipſin ſatis oblongam ſed ſpecie datam ſum definiturus, quæ pariter tribus illis obſervationibus ſatisfaciat: utroque enim caſu ex tribus obſervationibus orbita cometæ poterit aſſignari. His factis præter tres has obſervationes quartam maxime remotam contemplabor, inquiſiturus, utrum parabola an ellipſis aſſumta ad illam propius accedat: nimirum tam ex parabola quam ellipſi ad tempus quartæ obſervationis cometæ longitudinem ac latitudinem geocentricam computabo, & cum loco obſervato comparabo. Quodſi enim alteruter locus conſentiat, indicio id erit orbitam cometæ revera congruere cum ea, quæ hunc conſenſum produxerit: ſin autem neuter locus computatus cum obſervato conſpiret, ex ipſo diſſenſu vera orbitæ ſpecies, utrum ſit magis minusve oblonga quam aſſumta, colligi poterit; ſicque ſi orbita cometæ reperiatur ellipſis, ejus tempus periodicum ſatis prope innoteſcet.

§. VI.

Antequam autem inveſtigationem orbitæ cometæ ſuſcipiam, neceſſe erit motum cometæ cum in orbita parabolica, tum in ellipſi admodum oblonga ſpecie data attentius perpendere, atque ad calculum revocare. Sit igitur primum orbita cometæ

Fig. I. PML parabola, in cujus foco S poſitus ſit Sol: ſitque P vertex parabolæ ſeu perihelium & diſtantia perihelii a ſole PS ponatur $= a$; erit ſi ex S ad axem PS normalis SL ducatur, applicata SL æqualis ſemiſſi lateris recti, ideoque SL $= 2a$. Conſideretur

retur jam cometæ locus quicunque M; hinc ad focum ducatur recta MS, itemque ad SP normalis MK. Ponatur angulus PSM $= v$, qui eſt cometæ anomalia vera, ejusque diſtantia a ſole ſeu recta MS $= y$. Nunc ex natura parabolæ eſt SM $=$ PS $+$ PK $= 2$PS $-$ KS: unde fit KS $= 2$PS $-$ SM $= 2a - y$. At poſito ſinu toto $= 1$, erit coſ. PSM $=$ coſ. $v. = \frac{KS}{SM} = \frac{2a-y}{y}$; ideoque $y = \frac{2a}{1 + coſ. v.}$ Cum autem ſit coſ. $\frac{1}{2} v = \sqrt{\frac{1 + coſ. v.}{2}}$; erit $y = \frac{a}{(coſ. \frac{1}{2} v^2)}$ ideoque coſ. $\frac{1}{2} v = \sqrt{\frac{a}{y}}$. Ponatur tang. $\frac{1}{2} v = t$; erit $dv = \frac{2dt}{1+tt}$; & coſ. $\frac{1}{2} v = \frac{1}{\sqrt{(1+tt)}}$ hincque fit $y = a(1+tt)$; ſicque ex anomalia vera v diſtantia cometæ a ſole, & viciſſim ex hac diſtantia anomalia vera v invenitur. Ad motum autem cometæ cognoſcendum præcipue ſpectari debet area PSM, cujus elementum, ducto radio proximo Sm, eſt ſector infinite parvus MSm. Hujus area ob angulum MSm $= dv$ erit $= \frac{1}{2} yydv = aadt(1+tt)$; quare integrando habebitur area PSM $= aa(t + \frac{1}{3}t^3)$; cui tempus, quo cometa ſpatium PM in orbita ſua abſolvit, eſt proportionale. Vel cum ſit $t = \sqrt{\frac{y-a}{a}}$, erit ſubſtituto hoc valore area PSM $= \frac{1}{3} a(y + 2a)\sqrt{\frac{y-a}{a}}$.

§. VII.

Comparemus jam motum cometæ cum motu terræ, ut tempus, quo cometa datam aream conficit, in diebus menſurisque conſuetis exprimere poſſimus. Sit ſemiaxis transverſus ſeu diſtantia terræ a ſole media $= c$, ejusque latus rectum $=$ e; erit axis conjugatus $\sqrt{2ce}$; & area totius orbitæ telluris $= \frac{1}{2}\pi c\sqrt{2ce}$, denotante $1 : \pi$ rationem diametri ad peripheriam circuli;

culi; hanc autem aream totam terra circa ſolem abſolvit ſpatio anni ſiderei, ſeu 365 diebus 6 horis & 9 minutis, ſeu horis ac minutis ad partes diei decimales reductis tempore 365, 25625 dierum. Abſolvat autem cometa ſpatium ſeu aream PSM $= aa(t + \frac{1}{3}t^3)$ tempore n dierum, quoniam tempora ſunt ut areæ directe, & radices quadratæ ex lateribus rectis reciproce, habebitur hæc proportio:

$$365,25625 : n = \frac{\pi c\sqrt{2c}}{2} : \frac{a\sqrt{a}}{2}(t + \tfrac{1}{3}t^3):$$

quoniam orbitæ terræ latus rectum eſt $= c$, & orbitæ cometæ $= 4a$. Hinc ergo reperitur tempus, quo cometa ab P ad M progreditur $= n = \frac{365,25625 \cdot a\sqrt{a}}{\pi c\sqrt{2c}}(t + \frac{1}{3}t^3)$ dierum. Ponamus uti in tabulis aſtronomicis fieri ſolet, diſtantiam terræ a ſole mediam $c = 100000$; & quia eſt $\pi = 3,14159265$ coefficiens numericus ſequenti modo per logarithmos determinabitur.

	l c	= 5, 0 0 0 0 0 0 0
	l 2	= 0, 3 0 1 0 3 0 0
	l $2c$	= 5, 3 0 1 0 3 0 0
Ergo	l $\sqrt{2c}$	= 2, 6 5 0 5 1 5 0
add.	l c	= 5, 0 0 0 0 0 0 0
	l π	= 0, 4 9 7 1 4 9 9
		8, 1 4 7 6 6 4 9
ſubtr.	l 365, 25625	= 2, 5 6 2 5 9 7 6
	l denomin.	= 5, 5 8 5 0 6 7 3
	& ipſe denomin.	= 3 8 4 6 5 1, 5

Hanc obrem tempus, quo cometa anomaliam veram PSM $= v$, abſolvit erit $= \frac{a\sqrt{a}}{384651,5}(t + \frac{1}{3}t^3)$ dierum; ubi ad calculum expediendum notaſſe convenit numeri 384651, 5 logarithmum eſſe

eſſe $= 5,5850673$. Vel ob $t = \sqrt{\frac{y-a}{a}}$ erit idem tempus $= \frac{(y+2a)\sqrt{(y-a)}}{1153954,5}$ dierum; eſtque denominatoris 1153954,5 logarithmus $= 6,0621886$

§. VIII.

Quodſi jam diſtantia perihelii a Sole PS $= a$ fuerit cognita, ſimulque tempus, quo cometa in perihelio eſt verſatus, ſit datum, ad datum quodvis tempus vel ante vel poſt cometæ appulſum ad perihelium, ejus locus in orbita ſua poterit computari. Sit enim temporis intervallum inter momentum, quo cometa in perihelio exiſtit, & momentum temporis propoſitum $= n$ dierum, horis ſcilicet & minutis in partes diei decimales converſis; & habebitur iſta æquatio $n = \frac{a\sqrt{a}}{384651,5}(t+\frac{1}{3}t^3)$; quia autem $a\sqrt{a}$ in numeris datur, ponatur numerus $\frac{a\sqrt{a}}{384651,5} = N$. eritque $n = N\,(t+\frac{1}{3}t^3)$ ſeu $\frac{n}{N} = t+\frac{1}{3}t^3$. Pendet ergo inventio anomaliæ veræ ſeu anguli PSM propoſito tempori reſpondentis, cujus ſemiſſis tangens eſt $= t$, a reſolutione æquationis cubicæ $(t+\frac{1}{3}t^3) = \frac{n}{N}$. Quod cum opus eſſet vehementer moleſtum, pro ſingulis gradibus anomaliæ veræ v ſupputavi valores reſpondentes expreſſionis $(t+\frac{1}{3}t^3)$, poſito ſinu toto perpetuo $= 1$. Hos valores $(t+\frac{1}{3}t^3)$ ſingulis gradibus anomaliæ veræ v reſpondentes tabula annexa continet; cujus igitur uſus erit expeditus: quæratur enim numerus $\frac{n}{N}$ in hac tabula ſub titulo areæ parabolicæ ſeu formulæ $(t+\frac{1}{3}t^3)$ qui ſi ipſe reperiatur, anomalia vera reſpondens dabit angulum

PSM quæſitum, ſin autem numerus $\frac{n}{N}$ non exacte in hac columna reperiatur, inter proximos per interpolationem conſuetam anomalia vera reſpondens ad minuta & ſecunda concludetur; quem in finem differentias addidi. Cum autem plerumque ad logarithmum numeri $\frac{n}{N}$ perveniatur, tres feci columnas, quarum tertia logarithmos formulæ $t + \frac{1}{3}t^3$ exhibet, quo ex iis pari opera anomalia vera quæſita inveniri queat. Characteriſticam autem horum omnium logarithmorum denario auxi, ne numerorum unitate minorum logarithmi fierent negativi, quamobcauſam horum logarithmorum pariter ac logarithmorum ſinuum & tangentium characteriſticæ, in uſu denario ſunt minuendi: eo quod in tabulis logarithmus ſinus totius, cujus quippe unitatis logarithmus eſt $= 0$, ponitur $= 10,0000000$.

§. IX.

His pro parabola expeditis ponamus nunc orbitam, in qua cometa moveatur, eſſe ſectionem conicam quamcunque PML in cujus foco S poſitus ſit ſol, ac ponatur ut ante diſtantia perihelii a ſole $PS = a$. Ducatur ad axem ex foco S normalis SL, quæ erit ſemiſſis lateris recti, quæ cum non amplius, uti in parabola ſit $= 2a$, ponatur ea $SL = b$; ubi notandum eſt, ſi fuerit $b < 2a$ curvam fore ellipſin, ſin autem ſit $b > 2a$ eam fore hyperbolam, in applicatione autem ad cometas hæc recta b non multum a valore $2a$ diſcrepabit. Ex his erit orbitæ axis transverſus $= \frac{2aa}{2a-b}$; qui pro parabola ſi $b = 2a$, fit infinitus, pro ellipſi, ſi $b < 2a$ finitus & affirmativus, ſin autem $b > 2a$, pro hyperbola axis transverſus fit finitus quidem, at negativus. Verſetur jam cometa in loco M, ita ut ducta recta MS, ſit angulus

PSM

PSM ejus anomalia vera, ponatur recta MS $= y$, & angulus PSM $= v$. Ducta autem ex M ad axem PS perpendiculari MK, erit ex natura ſectionum conicarum MS $= y = a + \frac{(b-a)}{a}$. PK $= b - \frac{(b-a)}{a}$. KS; ideoque KS $= \frac{a(b-y)}{b-a}$. Cum autem $\frac{KS}{y} = \frac{KS}{SM}$ det coſinum anguli PSM $= v$, erit coſ. $v = \frac{a(b-y)}{y(b-a)}$ ideoque $y = \frac{ab}{a+(b-a)cof.v}$. Ponatur iterum tang. $\frac{1}{2}v = t$, ut ſit $dv = \frac{2dt}{1+tt}$ & coſ. $\frac{1}{2}v = \frac{1}{\sqrt{(1+tt)}}$ atque coſ. $v = \frac{1-tt}{1+tt}$; erit $y = \frac{ab(1+tt)}{b+(2a-b)tt}$. Quoniam vero hic ponimus orbitam quam minime a parabola diſcrepare, & eſſe ellipſin vehementer oblongam, ſit $b = 2a - \alpha a$; erit ellipſis axis tranſverſus $= \frac{2a}{\alpha}$; & cum ſit α numerus valde exiguus fiet $y = \frac{a(1+tt)(2-\alpha)}{2-\alpha(1-tt)} = a(1+tt)\ \frac{\alpha tt}{2}(1+tt) - \frac{\alpha\alpha tt}{4}(1-t^4)$ q. pr. ſeu $y = a(1+tt)\left(1 - \frac{\alpha tt}{2} - \frac{\alpha\alpha tt(1-tt)}{4}\right)$. Cognita autem anomalia vera v, diſtantia cometæ a ſole commodius invenitur per æquationem $y = \frac{a(2-\alpha)}{1+(1-\alpha)cof\,v}$, & viciſſim ſi diſtantia cometæ a ſole y fuerit data, erit coſ. $v = \frac{2a-y-\alpha a}{(1-\alpha)y}$.

§. X.

Illa autem expreſſio prope vera ad inveniendam aream PSM magis erit accommodata, cum enim elementum hujus areæ ſeu ſector MSm ſit $= \frac{1}{2}yydv$, erit hoc differentiale $= \frac{yydt}{1+tt} = aadt$ $(1+tt)\left(1 - \alpha tt - \frac{\alpha\alpha tt}{2} + \frac{3\alpha\alpha t^4}{4}\right) = aadt\ (1+tt - \alpha tt$

$-\alpha t^4$

$-\alpha t^4 - \frac{\alpha\alpha tt}{2} + \frac{\alpha\alpha t^4}{4} + \frac{3\alpha\alpha t^6}{4}$). Hinc integrando fiet area SPM $= aa\,(t + \frac{1}{3}t^3 - \frac{1}{3}\alpha t^3 - \frac{1}{5}\alpha t^5 - \frac{1}{6}\alpha\alpha t^3 + \frac{7}{20}\alpha\alpha t^5 + \frac{3}{28}\alpha\alpha t^7)$ quæ ut tempus præbeat per SM, dividi debet per radicem quadratam ex latere recto. Cum ergo latus rectum ſit $2b = 2a(2-\alpha)$ erit $\sqrt{2b} = \sqrt{a(4-2\alpha)}$ & $\frac{1}{\sqrt{2b}} = \frac{1}{\sqrt{a}}(\frac{1}{2} + \frac{\alpha}{8} + \frac{3\alpha\alpha}{64}$; unde ſit $\frac{ar.\,SMP}{\sqrt{2b}} = \frac{a\sqrt{a}}{2}\,(t + \frac{1}{3}t^3 + \alpha\,(\frac{1}{4}t - \frac{1}{4}t^3 - \frac{1}{5}t^5) + \alpha\alpha(\frac{3}{32}t - \frac{7}{32}t^3 + \frac{3}{28}t^7)$ hinc concluditur tempus, quo cometa ſpatium PM abſolvit, $= \frac{a\sqrt{a}}{384651,1}\,(t + \frac{1}{3}t^3 + \alpha\,(\frac{1}{4}t - \frac{1}{4}t^3 - \frac{1}{5}t^5) + \alpha\alpha(\frac{3}{32}t - \frac{7}{32}t^3 + \frac{3}{28}{}^7)$ dierum: ſeu $= \frac{a\sqrt{a}}{384651,5.\,\sqrt{(1-\frac{1}{2}\alpha)}}\,(t + \frac{1}{3}t^3 - \alpha(\frac{1}{3}t^3 + \frac{1}{5}t^5) - \alpha\alpha(\frac{1}{6}t^3 - \frac{1}{20}t^6 - \frac{3}{28}t^7))$ dierum. Cum autem ſit $y = \frac{a(2-\alpha)\,(1+tt)}{2-\alpha(1+tt)}$ erit $tt = \frac{(2-\alpha)\,(y-a)}{2a-\alpha(a+y)}$ & vero proxime $tt = \frac{y-a}{a} + \frac{\alpha y(y-a)}{2aa} + \frac{+\alpha\alpha y\;(yy-aa)}{4a^3}$. Sufficiet vero ad præſens inſtitutum terminos in quibus ineſt $\alpha\alpha$ negligere; propterea quod in hoc cometa, cujus curſum ſum determinaturus, quoad apparuit angulus PSM minor fuit recto, ideoque $t < 1$, & poteſtates ipſius t ſuperiores multo magis unitate minores. Erit ergo $tt = \frac{y-a}{a} + \frac{\alpha y(y-a)}{2aa}$ & $t = (1 + \frac{\alpha y}{4a})\sqrt{\frac{y-a}{a}}$; atque $t^4 = \frac{(y-a)^2}{aa} + \frac{\alpha y\,(y-a)^2}{a^3}$; unde cum tempus per PM ſit $= \frac{a\sqrt{a}}{384651,5\,\sqrt{(1-\frac{1}{2}\alpha)}}$ $(t + \frac{1}{3}t^3 - \alpha\,(\frac{1}{3}t^3 + \frac{1}{5}t^5))$ dierum, erit per y hoc idem tempus $= \frac{(a+\frac{1}{4}\alpha y)\sqrt{(y-a)}}{384651,5\,\sqrt{(1-\frac{1}{2}\alpha)}}\left(\frac{y+2a}{3a} - \frac{\alpha\,(y-a)(\alpha a+y)}{30aa}\right)$ dierum.

§. XI.

§. XI.

Si ergo præter a & α, atque tempus, quo cometa per perihelium tranſiit, detur vel anomalia vera v, cujus ſemiſſis tangens $= t$, vel diſtantia cometæ a ſole SM $= y$; poterit hinc tempus, quo cometa in hoc loco orbitæ ſuæ hæret, per expreſſiones datas aſſignari, ſiquidem α fuerit fractio minima. Sin autem tempus hoc fuerit datum puta $= n$ diebus, inventio anomaliæ veræ v aliquanto fit difficilior, quam in caſu parabolæ, requirit enim reſolutionem hujus æquationis $\frac{n}{N} = t + \frac{1}{3}t^3 - \alpha(\frac{1}{3}t^3 + \frac{1}{5}t^5)$, ſiquidem ponatur N pro numero $\frac{a\sqrt{a}}{384651,5\sqrt{(1-\frac{1}{2}\alpha)}}$. Interim tamen, quia α eſt fractio minima, hæc reſolutio ope tabulæ pro motu in parabola exhibitæ, effici poterit. Quæratur enim in tabula ſub columna $t + \frac{1}{3}t^3$ anomalia vera reſpondens, quæ ſit $= u$, & ponatur tang $\frac{1}{2}u = \theta$, erit $\frac{n}{N} = \theta + \frac{1}{3}\theta^3$. Jam anomalia vera pro caſu ellipſis, quæ ab hac inventa u non multum diſcrepabit, ſit $= u + \Phi = v$, ita ut ſit tang $\frac{1}{2}v =$ tang $(\frac{1}{2}u + \frac{1}{2}\Phi) = t$, unde ob Φ arcum minimum fit $t = \theta + \frac{1}{2}\Phi\,(1 + \theta\theta)$, ſubſtituatur hic valor in æquatione, & ob $\frac{n}{N} = \theta + \frac{1}{3}\theta^3$ reperietur $\Phi = \frac{2\alpha\theta^3(5+3\theta\theta)}{15(1+\theta\theta)^2}$, qui angulus, cum æqualis prodeat numero, invenietur ſi a logarithmo expreſſionis $\frac{2\alpha\theta^3(5+3\theta\theta)}{15(1+\theta\theta)^2}$, denario aucto ſubtrahatur 4, 6855749, numerus enim logarithmo reſiduo reſpondens dabit angulum Φ in minutis ſecundis expreſſum; cum autem ſit $\theta =$ tang $\frac{1}{2}u$, habebitur quoque $\Phi = \frac{2}{3}\alpha\,(\text{ſin}\,\frac{1}{2}u)^2$ tang $\frac{1}{2}u + \frac{2}{15}\alpha\,(\text{ſin}\,\frac{1}{2}u)^2$ ſin u; quæ expreſſio

facilius

facilius per logarithmos expeditur. Inventa autem anomalia vera v, reperitur diſtantia cometæ a ſole SM $= y$ ex hac æquatione $y = \frac{a(1-\alpha)}{1+(1-\alpha)\cos v}$; vel adhibitis approximationibus erit commodius $y = \frac{a}{(\cos \frac{1}{2} v)^2} - \frac{\alpha a (\tang \frac{1}{2} v)^2}{2(\cos \frac{1}{2} v)^2}$

§. XII.

Fig. II. Orbitam autem cometæ ex obſervationibus tribus aſſumtis ita determinabo, ut duas cometæ a ſole diſtantias a S & c una cum angulo intercepto a S c ſeu corda $a\,c$ inveniam, ex quibus deinceps orbitam cometæ, ſive ea ſit parabolica ſive elliptica, definiri oportet. Quamobrem ex datis duabus cometæ a ſole diſtantiis a S & c S una cum corda $a\,c$ ipſam orbitam cometæ ſive parabolicam ſive ellipticam ſpecie datam inveniri conveniet. Ponatur a S $= f$; c S $= g$ & corda $ac = k$; ac primo quidem pro parabola ſit diſtantia perihelii P a ſole S nempe PS $= a$; & anomalia vera ſeu angulus PS$a = v$, quæ ſunt duæ res ex datis definiendæ; ſit angulus aS$c = q$; anomalia vera PS$c = v + q$; hincque duæ ſequentes æquationes reſultabunt; $f = \frac{a}{(\cos \frac{1}{2} v)^2}$ & $g = \frac{a}{(\cos \frac{1}{2}(v+q))^2}$; ex quibus eliminata a fit $\sqrt{\frac{f}{g}} = \frac{\cos \frac{1}{2}(v+q)}{\cos \frac{1}{2} v} = \cos \frac{1}{2} q - \sin \frac{1}{2} q . \tang \frac{1}{2} v$: ideoque $\tang \frac{1}{2} v = \frac{\cos \frac{1}{2} q - \sqrt{\frac{f}{g}}}{\sin \frac{1}{2} g}$; unde angulus PS$a = v$, ideoque poſitio axis PS determinabitur. Quoniam vero angulus q non immediate datur, ſed demum ex triangulo a S c, in quo omnia latera ſunt cognita, erui debet;

debet; hanc eandem expreſſionem per ſola latera f, g & k exprimamus. Cum itaque ſit coſ. $q = \frac{ff + gg - kk}{2fg}$, erit coſ.

$\frac{1}{2}q = \sqrt{\frac{1 + \text{coſ.}\, q}{2}} = \sqrt{\frac{(f+g)^2 - kk}{4fg}} = \frac{\sqrt{(f+g+k)(f+g-k)}}{2\sqrt{fg}}$;

& ſin $\frac{1}{2}q = \sqrt{\frac{1 - \text{coſ.}\, q}{2}} = \sqrt{\frac{-(f-g)^2 + kk}{4fg}} = \frac{\sqrt{(f-g+k)(g-f+k)}}{2\sqrt{fg}}$.

Ex his naſcitur tang. $\frac{1}{2}v = \frac{\sqrt{(f+g+k)(f+g-k)} - 2f}{\sqrt{(f-g+k)(g-f+k)}}$;

& ſec. $\frac{1}{2}v = \frac{2\sqrt{(ff + fg - f\sqrt{(f+g+k)(f+g-k)})}}{\sqrt{(f-g+k)(g-f+k)}}$

$= \frac{\sqrt{2f(f+g+k)} - \sqrt{2f(f+g-k)}}{\sqrt{(f-g+k)(g-f+k)}}$.

ergo ſin $\frac{1}{2}v = \frac{\sqrt{(f+g+k)(f+g-k)} - 2f}{\sqrt{2f(f+g+k)} - \sqrt{2f(f+g-k)}} =$

$\frac{\frac{(g-f+k)}{2}\sqrt{\frac{f+g-k}{2}} - \frac{(f-g+k)}{2}\sqrt{\frac{f+g+k}{2}}}{k\sqrt{f}}$ & coſ. $\frac{1}{2}v =$

$\frac{\sqrt{(f-g+k)(g-f+k)}}{\sqrt{2f(f+g+k)} - \sqrt{2f(f+g-k)}}$. Cum igitur ſit $(\text{coſ.}\,\frac{1}{2}v)^2 =$

$\frac{(f-g+k)(g-f+k)}{4f(f+g) - 4f\sqrt{(f+g+k)(f+g-k)}}$; ob $a = f\,(\text{coſ.}\,\frac{1}{2}v)^2$,

erit $a = \frac{(f-g+k)(g-f+k)}{4(f+g) - 4\sqrt{(f+g+k)(f+g-k)}}$.

§. XIII.

Hinc tempus definiri poteſt, quod cometa ad ſpatium ac conficiendum impendit, per ſolas litteras f, g, & k; hoc enim tempore imprimis opus habeo ad calculum ſequentem expediendum. Sit tang. $\frac{1}{2}$ PSa $= r$; & tang. $\frac{1}{2}$ PSc $= s$; erit uti vi-

dimus $t = \frac{\sqrt{(f+g+k)(f+g-k)}-2f}{\sqrt{(f-g+k)(g-f+k)}}$ & $s = \frac{-\sqrt{(f+g+k)(f+g-k)}+2g}{\sqrt{(f-g+k)(g-f+k)}}$.
Cum ergo tempus a perihelio ad locum a ſit $= \frac{a\sqrt{a}}{m}(t + \frac{1}{3}t^3)$ & tempus a perihelio ad locum c ſit $= \frac{a\sqrt{a}}{m}(s + \frac{1}{3}s^3)$, poſito m pro numero 384651,5; erit tempus, quo cometa ſpatium ac abſolvit $= \frac{a\sqrt{a}}{m}(s + \frac{1}{3}s^3 - t - \frac{1}{3}t^3) = \frac{a\sqrt{a}}{m}(s-t)(1 + \frac{1}{3}s^5 + \frac{1}{3}st + \frac{1}{3}tt)$. Eſt vero $s - t = \frac{2(f+g)-2\sqrt{(f+g+k)(f+g-k)}}{\sqrt{(f-g+k)(g-f+k)}}$, $s + t = \frac{2(g-f)}{\sqrt{(f-g+k)(g-f+k)}}$; ex his oritur $ss + 1 = \frac{4g(f+g) - 4g\sqrt{(f+g+k)(f+g-k)}}{(f-g+k)(g-f+k)}$, $tt + 1 = \frac{4f(f+g) - 4\sqrt{(f+g+k)(f+g-k)}}{(f-g+k)(g-f+k)}$ & $1 + st = \frac{2(f+g) - 2\sqrt{(f+g+k)(f+g-k)}}{(f-g+k)(g-f+k)}\sqrt{(f+g+k)(f+g-k)}$. His additis fit $3 + ss + st + tt = \frac{2(+g) - 2\sqrt{(f+g+k)(f+g-k)}}{(f-g+k)(g-f+k)}$ $(2(g+f) + \sqrt{(f+g+k)(f+g-k)})$, ideoque habebitur $(s-t)(1 + \frac{1}{3}ss + \frac{1}{3}st + \frac{1}{3}tt) = \frac{4(f+g-\sqrt{(f+g+k)(f+g-k)})}{3(f-g+k)^{\frac{3}{2}}(g-f+k)^{\frac{3}{2}}}$ $(2(f+g) + \sqrt{(f+g+k)(f+g-k)})$. At eſt $a\sqrt{a} = \frac{(f-g+k)^{\frac{3}{2}}(g-f+k)^{\frac{3}{2}}}{8(f+g-\sqrt{(f+g+k)(f+g-k)})^{\frac{3}{2}}}$, quæ duæ expreſſiones ob $\sqrt{(f+g-\sqrt{(f+g+k)(f+g-k)})} = \sqrt{\frac{f+g+k}{2}} - \sqrt{\frac{f+g-k}{2}}$ in ſe mutuo ductæ dant $\frac{a\sqrt{a}}{m}(s-t)(1 + \frac{1}{3}ss + \frac{1}{3}st + \frac{1}{3}tt) = \frac{1}{6m}(2f + 2g + \sqrt{(f+g+k)(f+g-k)}$

$\sqrt{f} +$

$\frac{(\sqrt{f+g+k})}{2} - \frac{\sqrt{f+g-k})}{2} = \frac{\frac{(f+g+k)^{\frac{3}{2}}}{2} - \frac{(f+g-k)^{\frac{3}{2}}}{2}}{3m}$ quæ expreſſio dat tempus, quo cometa in orbita parabolica motus ſpatium ac conficit, in diebus expreſſum.

§. XIV.

Hæc eadem expreſſio facilius inveniri poteſt, ſi tempora a perihelio P ad loca a & c ex diſtantiis f & g ſolis definiantur, erit enim manente $m = 384651,5$ tempus quo cometa ex P ad a pertingit $= \frac{(2a+f)\sqrt{(f-a)}}{3m}$ & tempus, quo cometa ex P ad c provenit $= \frac{(2a+g)\sqrt{g-a)}}{3m}$ dierum (7). Cum autem ſit $a = \frac{kk-ff+2fg-gg}{4f+4g-4\sqrt{(f+g+k)(f+g-k)}}$ erit $f-a = \frac{5ff+2fg+gg-kk-4f\sqrt{(f+g+k)(f+g-k)}}{4f+4g-4\sqrt{(f+g+k)(f+g-k)}}$ & $\sqrt{(f-a)} = \frac{-2f+\sqrt{(f+g+k)(f+g-k)}}{\sqrt{2(f+g+k)}-(\sqrt{2(f+g-k)}}$ $= \frac{-(f-g+k)\sqrt{2(f+g+k)}+(g-f+k)\sqrt{2(f+g-k)}}{4k}$ & $\sqrt{(g-a)}$ $= \frac{2g-\sqrt{(f+g+k)(f+g-k)}}{\sqrt{2(f+g+k)}-\sqrt{2(f+g-k)}} = \frac{+(g-f+k)\sqrt{2(f+g+k)}-(f-g+k)}{4k}$ $\sqrt{2(f+g-k)}$. Quoniam igitur tempus ac eſt $= \frac{1}{3m}(2a(\sqrt{(g-a)}-\sqrt{(f-a)}) + g\sqrt{(g-a)} - f\sqrt{(f-a)})$ fiet $\sqrt{(g-a)} - \sqrt{(f-a)} = \frac{2f+2g-2\sqrt{(f+g+k)(f+g-k)}}{\sqrt{2(f+g+k)}-\sqrt{2(f+g-k)}}$ ideoque $2a(\sqrt{g-a)}-\sqrt{(f-a)}) = \frac{kk-ff+2fg-gg}{\sqrt{2(f+g+k)}-\sqrt{2(f+g-k)}}$ & $g\sqrt{(g-a)} - f\sqrt{(f-a)} = \frac{2ff+gg-(f+g)\sqrt{(f+g+k)(f+g-k)}}{\sqrt{2(f+g+k)}-\sqrt{2(f+gk-)}}$ Ergo tempus per $ac = \frac{1.}{3m} \frac{kk+(f+g)^2-(f+g)\sqrt{(f+g+k(f+g-k)}}{\sqrt{2(f+g+k)}-\sqrt{2(f+g-k)}}$ dierum, cujus fractionis ſi numerator & denominator per

$\sqrt{2}$

$\sqrt{2(f+g+k)} + \sqrt{2(f+g-k)}$ multiplicetur, prodibit tempus quo cometa ab a ad c progreditur $= \frac{1}{3m}\left(\frac{(f+g+k)^{\frac{3}{2}}}{2} - \frac{(f+g-k)^{\frac{3}{2}}}{2}\right)$ dierum $= \dfrac{\frac{(f+g+k)^{\frac{3}{2}}}{2} - \frac{(f+g-k)^{\frac{3}{2}}}{2}}{1153954,5}$ dierum quæ expreſſio uti ad calculum maxime eſt accommodata, ita etiam imprimis notata eſt digna, quod tempus, quo cometa in parabola latus datum ſpatium ac abſolvit, tum concinne ex ſolis diſtantiis a S & c S una cum corda ac poſſit exprimi.

§. XV.

Difficilius hæc expedientur, ſi curva, in qua cometa movetur, ponatur ellipſis ſpecie data. Manentibus autem $aS = f$, $cS = g$ & corda $ac = k$, quæ ſunt data, cum relatione inter SI & SP; quæ ſit uti ſupra poſuimus ut 1 ad $2-\alpha$. Ponatur ergo diſtantia perihelii a ſole $SP = a$; erit ſemi-latus rectum $SI = (2-\alpha)a$; ſit inſuper anomalia vera $PSa = v$, & poſito angulo $aSc = q$ erit anomalia vera $PSc = v+q$. Jam ex natura ellipſis erit $f = \frac{a(2-\alpha)}{1+(1-\alpha)\cos v}$ & $g = \frac{a(2-\alpha)}{1+(1-\alpha)\cos(v+q)}$ ideoque $\frac{f}{g} = \frac{1+(1-\alpha)\cos(v+q)}{1+(1-\alpha)\cos v} = \dfrac{\frac{1}{1-\alpha} + \cos v\cos q - \sin v\sin q}{\frac{1}{1-\alpha} + \cos v}$ ſeu

$\frac{f-g}{1-\alpha} = g\cos v.\cos q. - f\cos v. - g\sin v.\sin q.$ vel $g\sin q.\sin v. = g\cos q.\cos v. - f\cos v. + \frac{g-f}{1-\alpha}$. Quod ſi ſumatur hæc æquatio generalis $A\sin v = B\cos v + C$ fiet $\cos v = \frac{-BC + A\sqrt{(AA+BB-CC)}}{AA+BB}$ eſt que noſtro caſu $A = g\sin q$; $B = g\cos q - f$; & $C = \frac{g-f}{1-\alpha}$

hinc

hincque $AA + BB = ff - 2fg\cos v + gg = kk$. Hinc cum ſit $\sin \frac{1}{2} v = \frac{\sqrt{1 - \cos v}}{2}$ erit $\sin \frac{1}{2} v = \frac{\sqrt{(k+B)(k+C)} - \sqrt{(k-B)(k-C)}}{2k}$. Invento autem angulo $PSa = v$ fiet $a = \frac{f(1 + (1-\alpha)\cos v)}{2 - \alpha}$; & ſi loco coſ v valor inventus ſubſtituatur, vel ex binis æquationibus, ſuperioribus angulus v eliminetur, reperietur $(2-\alpha)\, a = b =$

$$\frac{4fg - (1-\alpha)^2 (f+g+k)(f+g-k)}{2(f+g) - 2\sqrt{\frac{(f+g+k)(f+g-k)((1-\alpha)k - f + g)((1-\alpha)k + f - g)}{(k-f+g)(k+f-g)}}}.$$

Quocirca formulis, quæ commodiores videbuntur, uti licebit ad rectam a & angulum v inveniendum. Quibus inventis erit tempus, quo cometa ab a ad c pervenit, in diebus expreſſum $=$

$$\frac{(a + \frac{1}{4}\alpha g)\sqrt{(g-a)}}{m\sqrt{(1-\frac{1}{2}\alpha)}}\left(\frac{2a+g}{3a} - \frac{\alpha(g-a)(4a+g)}{30aa}\right) - \frac{(a + \frac{1}{4}\alpha f)\sqrt{(f-a)}}{m\sqrt{(1-\frac{1}{2}\alpha)}}\left(\frac{2a+f}{3a} - \frac{\alpha(f-a)(4a+f)}{30aa}\right)$$

denotante $m = 384651,5$.

§. XVI.

Hos igitur ad orbitam cometæ menſe martio hujus anni obſervati determinandam accommodabo. Obſervationes autem, quibus ad hoc inſtitutum utar, mecum communicatæ ſunt a Celeberrimo Aſtronomo De-Liſle, quas ſumma cura in obſervatorio Petropolitano habuit. Tempora autem, quibus hæ obſervationes ſunt factæ, primo ad ſtilum veterem & meridianum Londinenſem commoditatis gratia reduxi, tum vero in tempus aſtronomicum medium converti: obſervatæ autem ſunt temporibus indicatis cometæ longitudines ac latitudines, uti ſequens tabella declarat.

Anno 1742.			Longit. Cometæ.			Latit. Cometæ Borealis.	
Febr. 25^d,	10^h,	0′	9^s,	24°,	13′	56°,	29′
27^d,	10^h,	13′	10,	3,	16	66,	41
28^d,	6,	49	10,	9,	3	70,	30
—	15,	8	10,	11,	56	71,	54
Mart. 1^d,	7,	57	10,	19,	12	74,	30
—	15,	10	10,	22,	13	75,	29
2^d,	7,	2	11,	1,	49	77,	24
—	7,	39	11,	2,	15	77,	28
3^d,	14,	43	11,	24,	4′	79,	30
4^d,	14,	34	0,	13,	20	79,	55
6^d,	13,	55	1,	12,	24	77,	46
—	14,	15	1,	12,	44	77,	37
15^d,	6,	45	2,	16,	18	65,	20
16^d,	6,	38	2,	17,	31	64,	16
17^d,	7,	50	2,	18,	52	63,	13

Factæ ergo ſunt hæ obſervationes trium hebdomadarum intervallo, quod etſi tempus ſatis eſt exiguum, tamen motus cometæ ſecundum longitudinem per *6* fere ſigna eſt abſolutus, ex quo celerrimo motu orbita cometæ ſatis exacte definiri poterit, cum errores obſervationum in orbitæ determinatione non ſolum non multiplicentur, ſed etiam tantopere diminuantur, ut prorſus evaneſcant.

§. XVII.

Ad ſingula hæc obſervationum tempora jam primum loca ſolis vel terræ exactiſſime definiri debent, cum enim motus cometæ ſit celerrimus, niſi loca terræ, ad quæ cometæ loca referuntur, ſint accuratiſſime cognita, ex minimis erroribus maximi errores in motum cometæ verum redundarent. Quamobrem ad inventionem locorum terræ relictis tabulis aſtronomicis malo ea motus telluris theoria uti, quam alia occaſione

ex

ex obſervationibus exquiſitiſſimis elicui. Reduxi autem hanc theoriam ad Calendarium Julianum & meridianum Londinenſem, atque anomalia terræ media ad tempora deſcripta ita ſe habet;

Anno	Anomalia media				Long. I * ♈			
1701	6ˢ,	13°,	17′,	50″	0ˢ,	29°,	0′,	30″
1721	6,	13,	10,	14	0,	29,	17,	10
1741	6,	13,	2,	38	0,	29,	33,	50
1761	6,	12,	55,	2	0,	29,	50,	30
1781	6,	12,	47,	26	1,	0,	7,	10
1801	6,	12,	39,	51	1,	0,	23,	50

Intelliguntur hic anni currentes, atque anomaliæ mediæ reſpondent meridiei ultimi diei anni præcedentes. Pro annis autem mediis motus terræ medius ex ſequenti tabella colligetur, pariter ac pro initio cujusque menſis.

Annis	Motus medius	Præc. Æquin
1	11ˢ, 29°, 44′, 50″	0°, 0′, 50″
2	11, 29, 29, 40	0, 1, 40
3	11, 29, 14, 30	0, 2, 30
4	11, 29, 58, 29	0, 3, 20
5	11, 29, 43, 19	0, 4, 10
6	11, 29, 28, 9	0, 5, 0
7	11, 29, 12, 59	0, 5, 50
8	11, 29, 56, 58	0, 6, 40
9	11, 29, 41, 48	0, 7, 30
10	11, 29, 26, 38	0, 8, 20
11	11, 29, 11, 28	0, 9, 10
12	11, 29, 55, 27	0, 10, 0
13	11, 29, 40, 17	0, 10, 50
14	11, 29, 25, 7	0, 11, 40
15	11, 29, 9, 57	0, 12, 30
16	11, 29, 53, 56	0, 13, 20
17	11, 29, 38, 46	0, 14, 10
18	11, 29, 23, 36	0, 15, 0
19	11, 29, 8, 26	0, 15, 50
20	11, 29, 52, 24	0, 16, 40

Menſe	Motus medius	Præc
Januar.	0ˢ, 0°, 0′, 0″	0″
Februar.	1, 0, 33, 14	4
Martio	1, 28, 9, 4	8
Aprili	2, 28, 42, 18	12
Majo	3, 28, 16, 24	17
Junio	4, 28, 49, 37	21
Julio	5, 28, 23, 43	25
Auguſto	6, 28, 56, 57	29
Septemb.	7, 29, 30, 11	34
Octobr.	8, 29, 4, 17	38
Novemb.	9, 29, 37, 30	42
Decembr.	10, 29, 11, 36	46

§. XVIII.

§. XVIII.

Motus diurnus medius & horarius terræ non differt a reliquis tabulis, quamobrem iſta tabulæ mediorum motuum partem hic prætermitto. Deinde longitudinem aphelii orbitæ terræ a prima ſtella arietis computatam conſtanter ſtatuo $8^s, 8^\circ, 25', 42''$, & poſita diſtantia terræ a Sole media $=$ 100000, orbitæ terræ excentricitatem inveni eſſe 1687, 3, hincque æquationem maximam $1^\circ, 56', 0''$. Ex his datis ad quod vis tempus locus terræ verus in ecliptica poterit definiri una cum ejus diſtantia a Sole, ſequenti modo. Ex tabula motuum mediorum colligatur ad propoſitum tempus anomalia terræ media, una cum longitudine primæ ſtellæ arietis; ſit anomalia terræ media inventa $= x$; tum quæratur anomalia *excentrica dicta* v per hanc æquationem $v = x - k$ ſin $(x - k$ ſin $(x - k$ ſin $(x - k$ ſin $(x$ &c.$))$ ubi eſt $k = \frac{16873}{1000000}$, & $lk = 8,2271923$ aucta characteriſtica denario. Occurrunt hic ſinus arcuum per k multiplicati, qui in arcus converti debent, id quod fit ſi a logarithmo ſinus ſubtrahatur hic logarithmus 6, 4583826 tum enim numerus reſiduo logarithmo reſpondens dabit arcum quæſitum in minutis ſecundis. Inventa anomalia excentrica v erit primum diſtantia terræ a Sole $= 100000\,(1 + k\,\text{coſ}(v))$; ac deinde anomalia vera $z = v - \text{tang.}\tfrac{1}{2}\alpha\,\text{ſin}\,v + (\text{tang.}\tfrac{1}{2}\alpha)^2\ \text{ſin}\ 2v$, exiſtente α arcu cujus ſinus eſt k. In anomaliæ veræ igitur inventione primo *a* logarithmo *ſin.* v ſubtrahatur 6,4583517; & a logarithmo ſin. $2\,v$, ſubtrahatur 8, 8331885; numerus priori logarithmo reſiduo reſpondens dat arcum $= 2$ tang. $\tfrac{1}{2}$ α ſin v, & numerus poſteriori logarithmo reſiduo reſpondens

dabit

dabit arcum $=$ (tang. $\frac{1}{2}\alpha)^2$ ſin $2v$, utrumque in minutis ſecundis expreſſum. Denique ad anomaliam veram z inventam addatur primo longitudo aphelii a $1 * \Upsilon = 8^s, 8^\circ 25', 42''$, & inſuper longitudo ipſius primæ ſtellæ arietis, ſicque prodibit longitudo terræ ab æquinoctio verno computata; quæ 6 ſignis ſive aucta ſive minuta dabit locum Solis verum in ecliptica.

§. XIX.

Quo uſus hujus regulæ facilius percipiatur ad meridies trium ultimarum obſervationum ſpeciminis loco terræ loca quæram; ſi enim ad meridiem cujusque diei habeatur locus terræ vel ſolis, pro quovis tempore intermedio per interpolationem idem inveniri poterit. Sit igitur primum propoſitum hoc tempus A. 1742. M. Martio, $15^d, 0^h, 0'$, ad quod anomalia media eruatur.

	Anomalia media	Long. $1 * \Upsilon$
A. 1741	$6^s, 13^\circ, 2'\ 28''$	$0^s, 29^\circ, 33'\ 50''$
Ann. 1	11, 29, 44, 50	50
M. Mart.	1, 28, 9, 4	8
15 d.	: 14, 47, 3	2
A. 1742. Mart. 15	8, 25, 43, 25	$0^s, 29^\circ, 34'\ 50$
Mart. 16	8, 26, 42, 33	0, 29, 34, 50
Mart. 17	8, 27, 41, 41	0, 29, 34, 50

Eſt ergo anomalia media x ſex ſignis major, ponatur $x = 6^s + s$, erit ſin $x = -$ ſin s, ideoque anomalia *excentrica* erit $v = x + k$ ſin $(s + k$ ſin $(s + k$ ſin s &c.$))$, unde calculus pro inventione anomaliæ excentricæ ita ſe habebit.

	Pro Mart. 15 d.	Pro Mart. 16 d.	Pro Mart. 17 d.
s =	85°,43′25″	86°,42′33″	87°,41′,41″
l ſin s =	9,9987853	9,9992793	9,9996449
ſubtr.	6,4583826	6,4583826	6,4583826
	3,5404027	3,5408967	3,5412623
k ſin s =	3471″	3474″	3477″
ſeu =	57′,51″	57′54″	57′, 57″
addatur ad s —	86°,41′, 16″	87′,40′,27″	88°, 39′38″
l ſin =	9,9992720	9,9996398	9,9998794
ſubtr.	6,4583826	6,4583826	6,4583826
	3,5408894	3,5412572	3,5414968
Arcus	3474″	3477″	3479,″
ſeu	57′, 54″	57′,57″	57′,59″
add. ad s	86°,41′, 19″	87°,40′, 30″	88° 39′,40″
Ergo v =	8^s,26°,41′19″	8^s,27°,40,′30″	8^s,28°,39′,40″

ſcilicet quia ulteriores operationes perpetuo eundem valorem eſſent exhibituræ pro k ſin ($s + k$ ſin ($s +$ &c.): poſt ſecundam calculus jam finiri poterit, ſicque expedite anomalia excentrica definitur, quod idem commodum perpetuo in ſimili calculo uſu veniet.

§. XX.

Cum jam diſtantia terræ a ſole ſit = 100000 + 100000 k coſ v; v vero ſit angulus duobus rectis major, tribus vero minor, ejus coſinus erit negativus, ideoque ſi ponatur $v = 6^s + s$, erit diſtantia terræ a ſole = 100000 − 100000 k coſ s: unde calculus ita inſtituetur.

Mart.

	Mart. 15 d.	Mart. 16 d.	Mart. 17 d.
$s =$	86°,41′,19″	87, 40, 30	88, 39′, 40
l coſ $s =$	8,7616457	8,6081811	8,3685824
l 100000 k =	3,2271923	3,2271923	3,2271923
	1,9888380	1,8353734	1,5957747
100000k coſ.s =	97, 46	68, 45	39, 42
ſubtr. ab	100000	100000	100000
Diſt. ♁ a ☉ =	99902,54	99931,55	99960,58
l diſt. ♁ a ☉ =	4,9995766	4,9997026	4,9998287

Ad anomaliam denique veram inveniendam, ob $v = 6^s + s$ erit ſin $v = -$ ſin s & ſin $2v =$ ſin $2s$, hincque anomalia vera $z = v + 2$ tang. $\frac{1}{2}\alpha$. ſin $s +$ (tang $\frac{1}{2}\alpha)^2$ ſin $2s$. Fiet ergo

$s =$	86°, 41′, 19″	87°, 40′, 30″	88°, 39′, 40″
$2s =$	173, 22, 38	175, 21, 0″	177°, 19, 20
$180 - 2s =$	6, 37, 22	4, 39, 0	2, 40, 40
l ſin $s =$	9,9992720	9,9996423	9,9998814
ſubtr.	6,4583517	6,4583517	6,4583517
	3,5409203	3,5412906	3,5415297
2tang $\frac{1}{2}\alpha$.ſin $s =$	3475	3478	3480
ſeu	57′,55″	57′58″	58′,0″
l ſin $2s =$	9,0619501	8,9088535	8,6676893
ſubtr.	8,8331885	8,8331885	8,8331885
	0,2287616	0,0756650	9,8345008
(tang$\frac{1}{2}\alpha)^2$ ſin $2s$	2″	1″	0″
add. ad $v =$	8^s,26°,41,19″	8,27°,40,30	8,28,39,40
Anomalia vera	8,27°,39,16	8,28,38,29	8,29,37,40
addantur	8, 8, 25,42	8, 8,25,42	8, 8,25,42
Præc. Æquin.	0,29,34,50	0,29,34,50	0,29,34,50
Longit. terræ.	6^s, 5,39,48	6, 6,39, 1	6, 7,38,12

§. XXI.

§. XXI.

Simili modo pro meridie ſingulorum reliquorum dierum, quibus obſervationes ſunt factæ, terræ loca in ecliptica, ejusque a ſole diſtantias computavi, quæ hic annectuntur.

St. v. Meridie Londini.	Longitudo terræ.	Long. diſt. terræ a ſole.	Diſtantia terræ a ſole.
Febr. d. 24	5^s, 16°, 48′, 17″	4, 9 9 7 2 3 4 0	9 9 3 6 5, 1 5
25	5, 17, 48, 9	4, 9 9 7 3 5 2 7	9 9 3 9 2, 3 1
26	5, 18, 47, 59	4, 9 9 7 4 7 2 2	9 9 4 1 9, 6 4
27	5, 19, 47, 48	4, 9 9 7 5 9 2 2	9 9 4 4 7, [illegible] 3
28	5, 20, 47, 35	4, 9 9 7 7 1 3 1	9 9 4 7 4, 8 0
Mart. d. 1	5, 21, 47, 20	4, 9 9 7 8 3 4 4	9 9 5 0 2, 6 0
2	5, 22, 47, 3	4, 9 9 7 9 5 6 3	9 9 5 3 0, 5 3
3	5, 23, 46, 44	4, 9 9 8 0 7 8 8	9 9 5 5 8, 6 1
4	5, 24, 46, 23	4, 9 9 8 2 0 1 8	9 9 5 8 6, 8 0
5	5, 25, 45, 59	4, 9 9 8 3 2 5 1	9 9 6 1 5, 1 0
6	5, 26, 45, 33	4, 9 9 8 4 4 9 1	9 9 6 4 3, 5 3
7	5, 27, 44, 55	4, 9 9 8 5 7 3 4	9 9 6 7 2, 0 5
8	5, 28, 44, 34	4, 9 9 8 6 9 8 0	9 9 7 0 0, 6 7
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
15	6, 5, 39, 48	4, 9 9 9 5 7 6 6	9 9 9 0 2, 5 4
16	6, 6, 39, 1	4, 9 9 9 7 0 2 6	9 9 9 3 1, 5 6
17	6, 7, 38, 12	4, 9 9 9 8 2 8 7	9 9 9 6 0, 5 8

§. XXII.

Si hæc terræ loca comparentur cum locis cometæ obſervatis, facile colligitur tempore quodam inter 3 & 4 diem martii ſolis & cometæ eandem fuiſſe longitudinem, ſeu cometam cum ſole in conjunctione fuiſſe; hocque mox poſt obſervationem Mart. 3^d, 14^h, 43′, ubi longitudo cometæ obſervata eſt 11^s,

. 24°,

24°, 4′, eveniſſe. Ad hoc enim tempus locus ſolis colligitur fuiſſe 11^s, 24°, 23′, 19″, ejusque motus horarius erat 2′, 29″. Motus cometæ horarius autem ſi cum præcedente comparetur, reperitur 42′, 8″, ſin autem cum obſervatione ſequente conferatur, erit is 48′, 28″: ex quo hoc ipſo obſervationis tempore motus cometæ horarius ſecundum longitudinem erat quaſi 45′, 18″. Ponamus ergo conjunctionem veram eveniſſe Mart. 3^d, 14^h, $43' + x$ hor. prodibit hoc tempore longitudo ſolis 11^s, 24°, 23′, $19'' + 149\,x''$, & longitudo cometæ 11^s, 24°, 4′, $+ 2718\,x''$, quæ longitudines ſibi æquales poſitæ dant $19', 19'' = 1159 = 2569\,x$; ideoque x horæ $= 27'$, ita ut conjunctio vera acciderit Mart. 3^d, 15^h, 10′, quo tempore ſolis & cometæ longitudo erat eadem 11^s, 24°, 24′, 26″, quæ etſi non fuit obſervata, tamen loco obſervationis haberi poteſt.

§. XXIII.

Hoc igitur conjunctionis tempore iſtud acquirimus commodum, quod reliquæ obſervationes non ſuppeditant, ut longitudo cometæ heliocentrica innotescat; quippe quæ vel eadem eſſe debet, quæ eſt longitudo terræ, vel ei èdiametro oppoſita. Martii ſcilicet 3^d, 15^h, 10′ longitudo cometæ heliocentrica erat vel 5^s, 24°, 24′, 26″ vel 11^s, 24°, 24′, 26″ prout cometa fuerit vel cis vel trans ſolem verſatus. Mox autem patebit cometam hoc tempore nobis propiorem fuiſſe ſole, atque adeo ex ſole eandem longitudinem tenuiſſe ac terram. Hanc obrem iſte cometæ locus quaſi obſervatus cum duobus aliis commodiſſime adhibebitur ad orbitam cometæ inveſtigandam; eligantur ergo præter hunc cometæ locum duæ aliæ obſervationes, quarum altera ante, altera vero poſt conjunctionem

D 3 ſint

ſint habitæ; quæ neque ſint nimis viçinæ, neque nimis remotæ. Si enim ſumerentur duæ obſervationes nimis propinquæ, error ex obſervationum aberratione oriundus fieret ſenſibilis: ſin autem eſſent nimis remotæ, curvatura orbitæ impedimentum afferret. His perpenſis tria ſequentia cometæ loca ad hoc inſtitutum videntur aptiſſima.

Tempore.	Long. Com.	Lat. Comet.	Loc. Solis.	Diſt. terrae a Sole.
Febr. 28^d, 6^h, $49'$	10^s, $9°$, $3'$	$70°$, $30'$	11^s, $21°$, $4'$, $33''$	99482,70
Mart. 3, 15, 10	11, 24, 24	79, 31'	11, 24, 24, 26	99576,55
Mart. 6, 14, 15	1, 12, 44	77, 37	11, 27, 20, 48	99660,46

Hic tempus inter obſervationem primam & mediam elapſum eſt 3^d, 8^h, $21'$ $=$ 3, 34791 dies, quod ponatur $= m$, Tempus vero inter obſervationem mediam & tertiam elapſum eſt 2^d, 23^h, $5'$ $=$ 2, 96180 dies, quod ponatur $= n$, a prima igitur ad ultimam obſervationem elapſum eſt tempus $= m + n =$ 6, 30971 dierum.

§. XXIV.

Repræſentet jam planities tabulæ planum eclipticæ, in quo Sol obtineat locum S: deſignentur tria terræ loca A, B & C
Fig. 3. tribus hisçe obſervationibus convenientia, & ductis rectis AS, BS, CS erit.

AS $=$ 99482, 70	lAS $=$ 4, 9977476
BS $=$ 99576, 55	lBS $=$ 4, 9981571
CS $=$ 99660, 46	lCS $=$ 4, 9985229

& ang. ASB $= 3°, 19', 53''$
ang. BSC $= 2°, 56', 22''$
ang. ASC $= 6°, 16', 15''$

Quoniam

Quoniam porro ipſa recta BS exhibet cometæ longitudinem in obſervatione media, ducantur rectæ $A\alpha$, & $C\gamma$, quæ deſignent cometæ longitudines obſervatas in obſervatione prima & ultima; quarum poſitio ita erit determinata, ut ſit:

ang. $SA\alpha = 42°, 1'$	ang. $AmB = 45°, 20', 53''$
ang. $SC\gamma = 45°, 24'$	ang. $CnB = 48°, 20', 22''$

in cometæ enim locis hic negligo minuta ſecunda, quia obſervationes non ultra minuta prima ſunt exactæ. Secant igitur rectæ $A\alpha$ & $C\gamma$ rectam BS in punctis m & n, quæ primum definiri debebunt, ex formulis hisce, quas reſolutio triangulorum ſuppeditat.

$$Sm = \frac{AS.\ \text{ſin}\ SA\alpha}{\text{ſin}\ AmB};\ Am = \frac{AS.\ \text{ſin}\ ASB}{\text{ſin}\ AmB}$$

$$Sn = \frac{CS.\ \text{ſin}\ SC\gamma}{\text{ſin}\ CnB};\ Cn = \frac{CS.\ \text{ſin}\ BSC}{\text{ſin}\ CnB}$$

calculus ergo ita ſe habebit

	l AS =	4,9977476	l CS =	4,9985229
	l ſin AmB =	9,8521072	l ſin CnB =	9,8733764
		5,1456404		5,1251465
add.	(l ſin $SA\alpha$ =	9,8256512	l ſin $SC\gamma$ =	9,8524959
	(l ſin ASB =	8,7642580	l ſin BSC =	8,7099518
	l Sm =	4,9712916	l Sn =	4,9776424
	l Am =	3,9098984	l Cn =	3,8350983
	Sm =	93603, 40	Sn =	94982, 25
	BS =	99576, 55	BS =	99576, 55
	Bm =	5973, 15	Bn =	4594, 30
	Am =	8126, 40	Cn =	6840, 66

& mn = 1378, 85

§. XXV.

§. XXV.

Si jam ponamus a p c eſſe portionem orbitæ cometæ, quam intervallo obſervationum aſſumtarum deſcripſerit, erit *a* ejus locus tempore obſervationis primæ, ex quo perpendiculum in eclipticam demiſſum *a* α in rectam A α incidit: & *c*, unde perpendiculum in C γ cadit, erit locus cometæ in tertia obſervatione; atque *p* erit ejus locus in obſervatione media, ſi quidem perpendiculum *p* π in rectam B S incidit. Quare cum areæ circa ſolem deſcriptæ ſint tempuribus proportionales, erit area a S *p* ad aream *p* S *c* ut *m* ad *n*: atque ſi orbita cometæ in planum eclipticæ projiciatur $\alpha\ \pi\ \gamma$. quia ipſa orbita ad projectionem datam tenet rationem, erit quoque area α S π ad aream γ S π ut *m* ad *n*. Perſpicuum autem eſt rationem ſectorum α S π & γ S π tam prope ad rationem triangulorum α S β & β S γ accedere, ut error ſit prorſus imperceptibilis: cum quia ratio inter has areas fere eſt ratio æqualitatis, tum quia anguli α S β, & β S γ, ut ex ſequentibus patebit, tam ſunt parvi, ut curvatura hanc convenientiam turbare nullo modo poſſit. Cum igitur triangulorum α S β & β S γ ratio ſit ut *m* ad *n* erit, quoque $\alpha\beta : \beta\gamma = m : n$ ſicque ratio ſegmentorum cordæ $\alpha\ \gamma$ per rectam S B factorum erit cognita: unde ſi innoteſceret punctum β in recta B S, per quod corda $\alpha\gamma$ tranſit, ſimul poſitio cordæ ſeu angulus B *b* γ definiri poſſet. Ponamus enim ang. B $\beta\ \gamma = \Phi$, erit in triangulo $\alpha\ m\ \beta$ ang. $\alpha\, m\, \beta = A\, m\, B$ & $m\ \alpha\ \beta = \Phi - A\, m\, B$; in triangulo vero $\beta\ n\ \gamma$ erit ang. $\beta\, n\, \gamma = C\, n\, B$, & $\beta\, \gamma\, n = 180^\circ - \Phi - B\, n\, C$. Hinc fiet

$\alpha\beta =$

$$\alpha\beta = \frac{\beta m.\ \text{ſin}\ AmB}{\text{ſin}\ (\phi - AmB)} \quad \&\ \beta\gamma = \frac{\beta n.\ \text{ſin}\ BnC}{\text{ſin}\ (\phi + BnC)}$$

quare cum ſit $\alpha\beta : \beta\gamma = m : n$ habebitur hæc æquatio $\frac{n.\beta m.\ \text{ſin}\ AmB}{\text{ſin}\ (\phi - AmB)} = \frac{m.\beta n.\ \text{ſin}\ BnC}{\text{ſin}(\phi + BnC)}$, ſeu ob ſin ($\phi$–A$m$B) = ſin ϕ. coſ. AmB — coſ. ϕ. ſin AmB, & ſin (ϕ+BnC) = ſin ϕ. coſ. BnC + coſ. ϕ. ſin BnC, habebitur utroque denominatore per coſ. ϕ diviſo hæc æquatio

$$\frac{n.\ \beta m}{tang.\phi.\ cot.\ AmB - 1} = \frac{m.\ \beta n}{tang.\phi.\ cot.\ BnC + 1},$$ unde oritur

$$tang.\phi = \frac{\frac{m}{n}.\frac{Bn}{Bm} + 1}{\frac{m}{n}.\frac{Bn}{Bm}\ cot.AmB - cot.BnC}$$; ſicque ex cognito puncto β

invenitur poſitio cordæ $\alpha\gamma$, hineque porro ob latitudines datas puncta a & c in orbita cometæ. Cum vero ſit

$m = 3{,}34791$ erit $l\,m = 0{,}5247738$
$n = 2{,}96180$ $\quad l\,n = 0{,}4715557$

ideoque $l\frac{m}{n} = 0{,}0532181$

§. XXVI.

Huc ergo reducta eſt quæſtio, ut definiatur punctum β in recta Bβ, ſicque reſolutio quæſtionis ad unicam quantitatem incognitam eſt revocata. Neque tamen convenit, ut alias ſieri ſolet, pro hac incognita characterem quempiam adhibere, eumque per æquationem determinare, propter expreſſiones maxime complicatas, ad quas perveniretur. Expediet ergo ex aliquot falſis pro Bβ aſſumtis valoribus, verum per regulam *falſi* eruere; ut evitentur litteræ indeterminatæ ejusmodi calculum trigonometricum plurimum perturbantes. Sumantur ergo

 pro

pro arbitrio aliquot valores pro intervallo Bβ; & cum pro quolibet conveniens angulus $\Phi = B\beta\gamma$ fuerit definitus, habebuntur duo cometæ loca vera *a* & *c*, quorum tam a ſole diſtantiæ a S, c S, quam angulus interceptus a S c ſeu corda *a c* aſſignari poterunt. Hinc ex præcedentibus præceptis, ſive orbita cometæ ſtatuatur parabola, ſive ellipſis perquam oblonga computetur tempus, quod cometa ad ſpatium *a c* abſolvendum per theoriam impendere debet; quod ſi conveniat cum tempore obſervato 6,30971 dierum, indicio id erit pro Bβ valorem verum eſſe aſſumtum, ſin autem diſcrepet, duabus pluribusve hypotheſibus inter ſe comparandis valor prope verus ipſius Bβ colligitur. Tum tribuantur denuo ipſi *Bβ* duo valores non multum ab invento diſſidentes, qui cum ambo non multum a veritate ſint diſſenſuri, ſatis exacte verus valor pro *Bβ* concludetur. Oportebit autem duplicem pro *Bβ* valorem hoc pacto inveniri, alterum pro hypotheſi, qua orbita cometæ ſtatuitur parabola, alterum, ſi orbita cometæ ſtatuatur ellipſis ſpecie data, quo deinceps his duabus hypotheſibus cum reliquis obſervationibus comparandis vera ellipſeos ſpecies, ideoque vera cometæ orbita poſſit deduci.

§. XXVII.

Tribuam igitur ipſi Bβ hos duos valores 8000 & 9000 quoniam per alia craſſiora tentamina jam cognovi verum valorem inter hos limites contineri; ac pro utroque primum angulum $\Phi = B\beta\gamma$ determinabo.

Sit

Sit ergo $B\beta$ =	8000,00	9000,00
ſubtr. Bm =	5973,15	6973,15
erit βm =	2026,85	3026,85
add. mn =	1378,85	1378,85
erit βn =	3405,70	4405,70
Jam ſumatur $l\beta$n =	3,5322064	3,6440149
ſubtr. $l\beta$m =	3,3068216	3,4809908
erit $l\,\frac{\beta n}{\beta m}$ =	0,2253848	0,1630241
add. $l\,\frac{m}{n}$ =	0,0532181	0,0532181
erit $l\,\frac{m.\beta n}{n.\beta m}$ =	0,2786029	0,2161422
add. l cot. AmB =	9,9947234	9,9947234
erit $l\,\frac{m.\beta n}{n.\beta m}$ cot.AmB =	0,2733263	0,2109656
Hinc fit $\frac{m.\beta n}{n.\beta m}$ =	1,899340	1,645289
Ergo fractionis Numerator	2,899340	2,645289
Porro $\frac{m.\beta n}{n.\beta m}$ cot. AmB =	1,876404	1,625420
ſubtr. cot. BnC =	0,889734	0,889734
erit fractionis denominator	0,986670	0,735686
A l numeratoris	10,4622992	10,4224731
ſubtr. l denominatoris	9,9941719	9,8666924
Erit l tang. Φ =	10,4681273	10,5557807
ideoque ang. $B\beta\gamma$ = Φ =	71°,12′,22″	74°,27′,29″
Hinc reliqui anguli $\alpha m\beta$ =	45, 20, 53	45, 20, 53
ita ſe habebunt, ut $A\alpha\beta$ =	25, 51, 29	29, 6, 36
ſit $\beta n\gamma$ =	48, 20, 22	48, 20, 22
$C\gamma\beta$ =	60, 27, 16	58, 12, 9

§. XXVIII.

Quoniam nunc in triangulis $\alpha m \beta$, $\beta n \gamma$ dantur omnes anguli & latera βm & βn definientur reliqua latera per has formulas.

$$\alpha\beta = \frac{\beta m .\, ſin\, \alpha m \beta}{ſin\, A \alpha \beta}\ ; \quad \alpha m = \frac{\beta m .\, ſin\, \phi}{ſin\, A \alpha \beta}$$

$$\beta\gamma = \frac{\beta n .\, ſin\, \beta n \gamma}{ſin\, C \gamma \beta}\ ; \quad \gamma n = \frac{\beta n .\, ſin\, \phi}{ſin\, C \gamma \beta}$$

Inventis vero αm & γn erit $A\alpha = Am + \alpha m$ & $C\gamma = Cn + n\gamma$: calculus igitur ſequenti modo inſtituetur.

A $l\,\beta m$ =	3,3068216	3,4809908
ſubtr. l ſin $A\alpha\beta$ =	9,6396291	9,6870721
	3,6671925	3,7939187
add. (l ſin $\alpha m \beta$	9,8521072	9,8521072
($l\ \phi$	9,9762048	9,9838222
erit ($l\ \alpha\beta$ =	3,5192997	3,6460259
($l\ \alpha m$ =	3,6433973	3,7777409
Erit ergo $\alpha\beta$ =	3305,98	4426,15
αm =	4399,44	5994,33
addatur Am =	8126,40	8126,40
erit $A\alpha$ =	12525,84	14120,73
Porro a $l\,\beta n$ =	3,5322064	3,6440149
ſubtr. l ſin $C\gamma\beta$ =	9,9395012	9,9293758
	3,5927052	3,7146391
addatur (l ſin $\beta n \gamma$ =	9,8733764	9,8733764
(l ſin ϕ =	9,9762048	9,9838222
erit ($l\ \beta\gamma$ =	3,4660816	3,5880155
($l\ \gamma n$ =	3,5689100	3,6984613
erit ergo $\beta\gamma$ =	2924,70	3827,71
γn =	3706,04	4994,15
addatur C n =	6840,66	6840,66
erit $C\gamma$ =	10546,70	11834,81

§. XXIX.

Ex latitudinibus nunc obſervatis deducentur rectæ aα & cγ. erit enim aα = Aα tang. latit. & cγ = Cγ tang. latit. eſt vero latitudo in prima obſervatione = 70°, 30′, & latitudo in tertia = 77°, 37′. Inventis porro aα & cγ, ſi ducatur *ak* rectæ $\alpha\gamma$ parallela, erit $ak = \alpha\beta + \beta\gamma$; & $ck = c\gamma - a\alpha$: tum igitur invenitur angulus *cak*, ob tang. $cak = \frac{ck}{ak}$; ex hoc que angulo porro corda $ac = \frac{c \cdot k}{ſin\ cak}$, calculus ergo ita inſtituatur.

Ad *l* A α =	4,0978068	4,1498571
addatur *l* tang. lat.	10,4508513	10,4508513
erit *l* a α =	4,5486581	4,6007084
& a α =	35371,88	39875,71
Ad *l* C γ =	4,0231158	4,0731605
addatur *l* tang. lat.	10,6584481	10,6584481
erit *l* c γ =	4,6815639	4,7316086
& c γ =	48035,68	53902,47
ſubtr. a α =	35371,88	39875,71
erit c k =	12663,80	14026,76
Porro eſt $\alpha\beta$ =	3305,98	4426,15
addatur $\beta\gamma$ =	2924,70	3872,71
erit $\alpha\gamma$ = a k =	6230,68	8298,86
Nunc a *l* c k =	4,1025640	4,1469572
ſubtr. *l* a k =	3,7945355	3,9190184
erit *l* tang. c a k =	10,3080285	10,2279388
& c a k =	63°,48′,9″	59°,23′,22″
Ab *l* c k =	4,1025640	4,1469572
ſubtr. *l* ſin c a k =	9,9529268	9,9348257
erit *l* a c =	4,1496372	4,2121315
& a c =	14113,511	16297,89

§. XXX.

§. XXX.

Ducantur rectæ $S\alpha$ & $S\gamma$, atque in cordam $\alpha\gamma$ productam ex S ducatur normalis SQ: erit in triangulo rectangulo βSQ latus $S\beta = SB - B\beta$ cum angulis datum, ergo habebitur $SQ = S\beta$ ſin Φ, $\beta Q = S\beta$ coſ. Φ. Hinc erit $Q\alpha = Q\beta - \alpha\beta$ & $Q\gamma = Q\beta + \beta\gamma$: atque $\frac{QS}{Q\alpha}$ = tang. $S\alpha Q$, & $\frac{QS}{Q\gamma}$ = tang. $S\gamma Q$, quibus angulis cognitis erit $Sa = \frac{QS}{ſin.\ S\alpha Q}$ & $S\gamma = \frac{QS}{ſin.\ S\gamma Q}$: hæc ad computum revocentur:

a BS =	99576,55	99576,55
ſubtr. Bβ =	8000,00	9000,00
erit Sβ =	91576,55	90576,55
ſumatur *l* Sβ =	4,9617843	4,9570157
add. *l* ſin Φ =	9,9762048	9,9838222
add. *l* coſ.Φ =	9,5080779	9,4280436
l SQ =	4,9379891	4,9408379
l βQ =	4,4698622	4,3850593
Ergo βQ =	29502,73	24269,41
ſubtr. αβ =	3305,98	4426,15
addatur βγ =	2924,70	3872,71
αQ =	26196,75	19843,26
γQ =	32427,43	28142,12
Nunc a *l* QS =	4,9379891	4,9408379
ſubtr. *l* Qα =	4,4182474	4,2976130
ſubtr. *l* Qγ =	4,5109126	4,4493568
erit *l* tang. SαQ =	10,5197417	10,6432249
l tang. SγQ =	10,4270765	10,4914811
ergo ang. SαQ =	73°,11′,11″	77°,11′,21″
ergo ang. SγQ =	69 ,29, 31	72 , 7 ,33

Jam

Jam a l S Q =	4,9379891	4,9408379
ſubtr. (l ſin SαQ =	9,9810257	9,9890524
(l ſin SγQ =	9,9715648	9,9785151
l S α =	4,9569634	4,9517855
l S γ =	4,9664243	4,9623228

§. XXXI.

Inventis Sα & Sγ quoniam perpendicula $a\alpha$, $c\gamma$ ſunt data, erit tang. $aS\alpha = \frac{a\alpha}{S\alpha}$ & tang. $cS\gamma = \frac{c\gamma}{S\gamma}$, ſicque prodeunt latitudines heliocentricæ $aS\alpha$ & $cS\gamma$: hincque reperiuntur ipſæ cometæ diſtantiæ a ſole Sa & Sc ex his formulis : $Sa = \frac{a\alpha}{ſin.\ aS\alpha}$ & $Sc = \frac{c\gamma}{ſin.\ cS\gamma}$.

A b l a α =	4,5486581	4,6007084
ſubtr. l S α =	4,9569634	4,9517855
l tang. a S α =	9,5916947	9,6489229
ergo a S α =	21°,20′,2″	21°, 1′,0″
a l a α =	4,5486581	4,6007084
ſubtr. l ſin a S α =	9,5608653	9,6095969
l S a =	4,9877928	4,9911115
& S a =	97228,33	97974,17
ſimili modo a l c γ =	4,6815639	4,7316086
ſubtr. l S γ =	4,9664243	4,9623228
l tang. c S γ =	9,7151396	9,7692858
& ang. c S γ =	27°,25′,40″	30°,27′, 1″
nunc a l c γ =	4,6815639	4,7316086
ſubtr. l ſin c S γ =	9,6633524	9,7048284
l S c =	5,0182115	5,0267802
& S c =	104282,54	106360,48

§. XXXII.

Inventis nunc duabus cometæ a ſole diſtantiis Sa & Sc, una cum corda ac definiri poterit tempus, quod ad hoc ſpatium abſolvendum cum in hypotheſi orbitæ cometæ parabolica, tum elliptica requiritur: ac primo quidem ſi orbita cometæ ſtatuatur parabola, poſitis $Sa = f$: $Sc = g$ & $ac = k$, erit tempus per $ac = \frac{\left(\frac{f+g+k}{2}\right)^{\frac{3}{2}} - \left(\frac{f+g-k}{2}\right)^{\frac{3}{2}}}{3m}$ dierum, exiſtente $3m = 1153954,5$ & $l\,3m = 6,0621886$.

addantur $f = Sa =$	97228,33	97974,17
$g = Sc =$	104282,54	106360,48
$k = ac =$	14113,58	16297,89
erit $f+g+k =$	215624,45	220632,54
& $\frac{f+g+k}{2} =$	107812,22	110316,27
ſubtr. $k =$	14113,58	16297,89
erit $\frac{f+g-k}{2} =$	93693,64	94018,38
Nunc eſt $l\,\frac{f+g+k}{2} =$	5,0326680	5,0426396
addatur ſemiſſis $=$	2,5163340	2,5213198
erit $l\left(\frac{f+g+k}{2}\right)^{\frac{3}{2}} =$	7,5490020	7,5639594
ſubtrahatur $l\,3m =$	6,0621886	6,0621886
l Partis prioris	1,4868134	1,5017708
Porro $l\,\frac{f+g-k}{2} =$	4,9717333	4,9732128
add. ſemiſſis	2,4858666	2,4866064
$l\left(\frac{f+g-k}{2}\right)^{\frac{3}{2}} =$	7,4575999	7,4598192
ſubtr. $l\,3m$	6,0621886	6,0621886
l part. poſt. $=$	1,3954113	1,3976306

Ergo

Ergo pars prior .	30,67704	31,75198
pars poſterior =	24,85486	24,98219
Tempus per ac in diebus =	5,82218	6,70979

§. XXXIII.

Cum igitur tempus inter hæc cometæ loca præterlapſum ſit 6,30971 dierum, valor $B\beta = 9000$ tempus præbet nimis magnum, alterque valor $B\beta = 8000$ tempus præbet nimis parvum, ſi quidem orbita cometæ ponatur parabola: ex quibus perſpicuum eſt, verum ipſius $B\beta$ valorem intra limites 8000 & 9000 contineri.

Subtrahamus ergo a tempore majore 6,76979
tempus minus 5,82218

quia hoc temporis augmentum 0,94761
oritur ab incremento 1000, quod valori $B\beta = 8000$ eſt adjectum, videamus quantum incrementum ad eundem valorum $B\beta = 8000$ addi debeat, ut prodeat tempus obſervatum
6,30971, quod tempus ex hoc valore ortum
5,82218 ſuperat

0,48753 exceſſu: inſtituatur ergo iſta analogia

$$0,94761 : 1000 = 0,48753 : 514$$

ex qua concluditur valor ipſius $B\beta$ fore circiter $8000 + 514 = 8514$. Hic autem valor tantum prope eſt verus, quia differentia inter valores 8000 & 9000 pro $B\beta$ aſſumtos nimis eſt magna, quam ut temporum incrementa incrementis ipſius $B\beta$ proportionalia ſtatui posſint. Propius vero ad valorem ipſius $B\beta$ pervenire poſſemus, ſi unum duosve valores medios pro $B\beta$ aſſumtos ſimili modo evolveremus.

§. XXXIV.

Videamus autem nunc cujusmodi tempora hi duo pro $B\beta$ aſſumti valores præbeant, ſi orbita cometæ ponatur ellipſis ſpecie data; ac ponamus eſſe ſemilatus rectum b ad diſtantiam perihelii a ſole a ut $2 - \frac{1}{50}$ ad 1, ita ut ſit $\alpha = \frac{1}{50}$. Ponatur cum dentur diſtantia $Sa = f$, $Sc = g$, & corda $ac = k$, hincque angulus $aSc = q$; anomalia vera loci $a = v$ ſitque $B = g \cos. q - f$, & $C = \frac{g-f}{1-\alpha} = \frac{g-f}{49}.50$; eritque ſin. $\frac{1}{2} v = \sqrt{(\frac{1}{2} + \frac{B}{2k})(\frac{1}{2} + \frac{C}{2k})} - \sqrt{(\frac{1}{2} - \frac{B}{2k})(\frac{1}{2} - \frac{C}{2k})}$, & invento angulo v orietur $a = \frac{f}{2-\alpha} + \frac{(1-\alpha) f \cos. v}{2-\alpha} = \frac{50 f}{99} + \frac{49 f \cos. v}{99}$ Ad angulum vero $q = aSc$ inveniendum, erit cos. $q = \frac{ff + gg - kk}{2fg}$ hincque $B = \frac{(g-k)(g+k)}{2f} - \frac{f}{2}$. ex quibus formulis tam angulus v, quam diſtantia a ſequenti modo eruentur.

Cum ſit $g =$	104282,54	106360,48
& $k =$	14113,58	16297,89
erit $g - k =$	90168,96	90062,59
& $g + k =$	118396,12	122658,37
& $\frac{g+k}{2} =$	59198,06	61329,18
Tum eſt $f =$	97228,33	97974,17
ſumatur $l\ \frac{g+k}{2} =$	4,7723074	4,7876672
add. $l(g-k) =$	4,9550570	4,9545444
	9,7273644	9,7422116
ſubtr. $lf =$	4,9877929	4,9911115
	4,7395715	4,7511001
ergo $\frac{(g-k)(g+k)}{2f} =$	54899,90	56376,76
ſubtr. $\frac{1}{2} f =$	48614,16	48987,08

erit

erit B =	6285,74	7389,68
at $2k$ =	28227,16	32595,78
A l B =	3,7983564	3,8686256
ſubtr. $l\,2k$ =	4,4506671	4,5131614
$l\ \frac{B}{2k}$ =	9,3476893	9,3554642
ergo $\frac{B}{2k}$ =	0,222684	0,226707
$\frac{1}{2}$ =	0,5	0,5
& $\frac{1}{2}+\frac{B}{2k}$ =	0,722684	0,726707
$\frac{1}{2}-\frac{B}{2k}$ =	0,277315	0,273292
Porro eſt g =	104282,54	106360,48
f =	97228,33	97974,17
$g-f$ =	7054,21	8386,31
& C $= \frac{50}{49}(g-f)$ =	7198,17	8557,46
l C =	3,8572221	3,9323448
ſubtr. $l\,2k$ =	4,4506671	4,5131614
$l\ \frac{C}{2k}$ =	9,4065550	9,4191834
$\frac{C}{2k}$ =	0,255008	0,262533
ergo $\frac{1}{2}+\frac{C}{2k}$ =	0,755008	0,762533
$\frac{1}{2}-\frac{C}{2k}$ =	0,244991	0,237466
add. $l\left(\frac{1}{2}+\frac{B}{2k}\right)$ =	9,8589484	9,8613594
add. $l\left(\frac{1}{2}+\frac{C}{2k}\right)$ =	9,8779516	9,8822586
	19,7369000	19,7436180
$l\sqrt{\left(\frac{1}{2}+\frac{B}{2k}\right)\left(\frac{1}{2}+\frac{C}{2k}\right)}$ =	9,8684500	9,8718090
ergo $\sqrt{\left(\frac{1}{2}+\frac{B}{2k}\right)\left(\frac{1}{2}+\frac{C}{2k}\right)}$ =	0,738669	0,744405

$l\left(\frac{1}{2}-\frac{B}{2k}\right) =$	9,4429733	9,4366269
$l\left(\frac{1}{2}-\frac{C}{2k}\right) =$	9,3891501	9,3756015
	18,8321234	18,8122282
$l\sqrt{\left(\frac{1}{2}-\frac{B}{2k}\right)\left(\frac{1}{2}-\frac{C}{2k}\right)} =$	9,4160617	9,4061141
ergo $\sqrt{\left(\frac{1}{2}-\frac{B}{2k}\right)\left(\frac{1}{2}-\frac{C}{2k}\right)} =$	0,260652	0,254750
a $\sqrt{\left(\frac{1}{2}+\frac{B}{2k}\right)\left(\frac{1}{2}+\frac{C}{2k}\right)} =$	0,738669	0,744405
ſin $\frac{1}{2}$ v =	0,478017	0,489655
Hinc $\frac{1}{2}$ v =	28°,33′,21″½	29°,19′,4½″
& v =	57°,6′,43″	58°,38′,9″
Deinde eſt f =	97228,33	97974,17
ergo $\frac{50}{99}$ f =	49105,23	49481,91
& $\frac{49}{99}$ f =	48123,12	48491,27
l $\frac{49}{99}$ f =	4,6823538	4,6856635
add. l coſ. v =	9,7347993	9,7164005
	4,4171531	4,4020640
Ergo $\frac{49}{99}$ f coſ. v =	26130,82	25238,53
add. $\frac{50}{99}$ f =	49105,23	49481,91
erit a =	75236,05	74720,44
& l a =	4,8764260	4,8734394
Deinde cum ſit B =	6285,74	7389,68
addatur f =	97228,33	97974,17
erit B + f =	103514,07	105363,85
A l (B + f) =	5,0149994	5,0226917
ſubtr. l g =	5,0182115	5,0267803
erit $l\frac{B+f}{g} = l$ coſ. q =	9,9967879	9,9959114
& q = a S c =	6 ,57′,36″	7°,50′,59″

§. XXXV.

§. XXXV.

Inventis anomaliis veris $PS\alpha$, PSc cum diſtantia $PS = a$, commodiſſime tempus, quod ad arcum ac deſcribendum impenditur, reperietur, ſi ſeorſim tempora, quibus cometa a perihelio P ad loca a & c pervenit, quærantur, & minus a majore ſubtrahatur. Quodſi autem ponatur tangens $\frac{1}{2}$ $PSa = t$. erit tempus per $Pa = \frac{a\sqrt{a}}{384651,5\sqrt{\frac{99}{100}}}(t + \frac{1}{3}t^3 - \frac{1}{3.50}t^3 - \frac{1}{5.50}t^5 - \frac{1}{6.50.50}t^3 + \frac{1}{20.50.50}t^5)$ dierum, quæ eadem expreſſio dabit tempus per Pc, ſi loco t ſubſtituatur tangens $\frac{1}{2}$ PSc. Calculus ergo ſequenti modo inſtituetur, poſito brevitatis ergo $\frac{a\sqrt{a}}{384651,5\sqrt{\frac{99}{100}}} = N$ & $t + \frac{1}{3}t^3 - \frac{1}{3.50}t^3 - \frac{1}{5.50}t^5 - \frac{1}{6.50.50}t^3 + \frac{1}{20.50.50}t^5 + \&c. = T$, ut ſit tempus $= NT$.

Tab. I. Fig. 2.

Cum ſit $l\ a$	=	4,8764260	4,8734394
add. $l\sqrt{a}$	—	2,4382130	2,4367197
erit $l\ a\sqrt{a}$	=	7,3146390	7,3101591
add. $l\sqrt{\frac{100}{99}}$	=	0,0021824	0,0021824
		7,3168214	7,3123415
ſubtr. l 384651, 5		5,5850673	5,5850673
l N	=	1,7317541	1,7272742
nunc eſt PSa	=	57°, 6′, 43″	58°, 38′, 9″
& $\frac{1}{2}$ PSa	=	28, 33, 21½	29, 19, 4½
ergo $l\ t$	=	9,7357755	9,7494155
$l\ t^3$	=	9,2073265	9,2482465
$l\ t^5$	=	8,6788775	8,7470775
hinc t	=	0,5442213	0,5615851
t^3	=	0,1611857	0,1771114
t^5	=	0,0477395	0,0558570

 add.

add. t =	0,5442213	0,5615851
$\frac{1}{3}$ t^3 =	0,0537286	0,0590371
	0,5979499	0,6206222
ſubtr. $\frac{1}{3.50}$ t^3 =	0,0010745	0,0011807
	0,5968754	0,6194415
ſubtr. $\frac{1}{5.50}$ t^5 =	0,0001909	0,0002234
	0,5966845	0,6192181
ſubtr. $\frac{1}{6.50.50}$ t^3 =	0,0000107	0,0000118
	0,5966738	0,6192063
add. $\frac{1}{20.50.50}$ t^5 =	9	11
erit T =	0,5966747	0,6192074
ergo lT =	9,7757375	9,7918361
add. lN =	1,7317541	1,7272742
	1,5074916	1,5191103
Tempus ab P ad a =	32,17300	33,04534 dies
Porro ad $\frac{1}{2}$ P S a =	28°,33′,21½″	29°,19′,4½″
add. $\frac{1}{2}$ a S c =	3 ,28 ,48	3 ,55 ,29½
erit $\frac{1}{2}$ P S c =	32°, 2′, 9″½	33°,14′,34″
erit ergo $l\,t$ =	9,7963958	9,8165018
$l\,t^3$ =	9,3891874	9,4495054
$l\,t^5$ =	8,9819790	9,0825090
hinc eſt t =	0,6257428	0,6553930
t^3 =	0,2450121	0,2815175
t^5 =	0,0959354	0,1209230
ad t =	0,6257428	0,6553930
add. $\frac{1}{3}$ t^3 =	0,0816374	0,0938392
	0,7073802	0,7492322
ſubtr. $\frac{1}{3.50}$ t^3 =	0,0016327	0,0018767
	0,7057475	0,7473555
ſubtr. $\frac{1}{5.50}$ t^5 =	3837	4837

ſubtr.

	0,7053638	0,7468718
ſubtr. $\frac{1}{6.50.50} t^3$ =	163	187
	0,7053475	0,7468531
ſubtr. $\frac{1}{20.50.50} t^5$ =	19	24
T =	0,7053494	0,7468555
ergo *l* T =	9,8484042	9,8732366
add. *l* N =	1,7317541	1,7272742
	1,5801583	1,6005108
Tempus a P ad c =	38,03280	39,85757
ſubtr. tempus a P ad a =	32,17300	33,04534
Tempus ab *a* ad *c*	5,85980	6,81223

§. XXXVI.

Cum igitur tempus, quo cometa ex *a* in *c* perveniſſe eſt obſervatus ſit 6,30971 dierum, inter valores binos pro Bβ aſſumtos ejus valor verus continebitur, eritque ideo major quam 8000 & minor quam 9000. Dum autem valor ipſius Bβ, qui poſitus erat 8000, millenario augebatur, tempus per ſpatium *a c* increvit diei parte 0,95243. Tempus autem ex hypotheſi Bβ = 8000 ortum deficit a tempore obſervato parte 0,44991; erit ergo proxime ut 95243 ad 1000 ita 44991 ad augmentum valori Bβ = 8000 addendum, ut ejus valor verus obtineatur; quare per regulam auream hoc augmentum erit = 472; ideoque Bβ = 8472. Qui valor autem pariter ac præcedens in hypotheſi parabolæ inventus tantum prope eſt verus, quoniam tempora per *a c* non in ratione valorum pro Bβ aſſumtorum creſcunt, ſed in ratione majore, ex quo uterque valor verus pro Bβ aliquanto minor erit, quam hoc tentamine craſ-

ſiore

ſiore eſt inventus. Quamobrem quo ad veritatem propius, & quidem tam prope, quam circumſtantiæ permittunt, accedamus, pro $B\beta$ iterum geminum valorem, alterum 8400, alterumque 8500 aſſumamus, & calculum pari modo inſtituamus.

§. XXXVII.

Sit igitur $B\beta$	=	8400,00	8500,00
ſubtr. Bm	=	5973,15	5973,15
erit βm	=	2426,85	2526,85
addatur mn	=	1378,85	1378,85
fiet βn	=	3805,70	3905,70
Tam ſumatur $l\beta n$	=	3,5804345	3,5916989
ſubtr. $l\beta m$	=	3,3850429	3,4025795
erit $l\,\frac{\beta n}{\beta m}$	=	0,1953916	0,1891194
add. $l\,\frac{m}{n}$	=	0,0532181	0,0532181
$l\,\frac{m.\ \beta n}{n.\ \beta m}$	=	0,2486097	0,2423375
add. l cot. AmB	=	9,9947234	9,9947234
erit $l\,\frac{m.\ \beta n}{n.\ \beta m}$ cot. AmB	=	0,2433331	0,2370609
Ergo $\frac{m.\ \beta n}{n.\ \beta m}$	=	1,772596	1,747179
Numerator		2,772596	2,747179
Porro $\frac{m.\ \beta n}{n.\ \beta m}$ cot. AmB	=	1,751190	1,726080
ſubtr. cot. BnC	=	0,889734	0,889734
Denominator		0,861456	0,836346
Ergo l Numerator.		10,4428865	10,4388870
l Denominat.		9,9352331	9,9223860

erit

erit l tang. φ =	10,5076534	10,5165010
ideoque ang. $B\beta\gamma$ =	72°,44′,23″	73°, 4′,4″
Ergo ob $\alpha m \beta$ =	45 ,20,53	45 ,20,53
erit $A\alpha\beta$ =	27 ,23,30	27 ,43,11
& ob $\beta n \gamma$ =	48 ,20,22	48 ,20,22
& $C\gamma\beta$ =	58 ,55,15	58 ,35,34
Jam a l βm =	3,3850429	3,4025795
ſubtr. l ſin $A\alpha\beta$ =	9,6628245	9,6675900
	3,7222184	3,7349895
add. { l ſin $\alpha m \beta$ =	9,8521072	9,8521072
add. { l ſin φ =	9,9799883	9,9807530
l $\alpha\beta$ =	3,5743256	3,5870967
l αm =	3,7022067	3,7157425
Ergo $\alpha\beta$ =	3752,54	3864,53
αm =	5037,40	5196,88
add. Am =	8126,40	8126,40
erit $A\alpha$ =	13163,80	13323,28
Porro a l βn =	3,5804345	3,5916989
ſubtr. l ſin $C\gamma\beta$ =	9,9327044	9,9311959
	3,6477301	3,6605030
add. { l ſin. $\beta n \gamma$	9,8733764	9,8733764
add. { l ſin. φ	9,9799883	9,9807530
l $\beta\gamma$ =	3,5211065	3,5338794
l γn =	3,6277184	3,6412560
ergo $\beta\gamma$ =	3319,76	3418,84
γn =	4243,44	4377,70
add. Cn =	6840,66	6840,66
erit $C\gamma$ =	11084,10	11218,36
Ad l $A\alpha$ =	4,1193813	4,1246111
addatur l tang. lat.	10,4508513	10,4508513
l $a\alpha$ =	4,5702326	4,5754624
& $a\alpha$ =	37173,43	37623,79

Ad $l\,C\gamma$ =	4,0447004	4,0499285
addatur l tang. lat.	10,6584481	10,6584481
erit $l\,c\gamma$ =	4,7031485	4,7083766
& $c\gamma$ =	50483,38	51094,80
ſubtr. $a\alpha$ =	37173,43	37623,79
erit ck =	13309,95	13471,01
Porro eſt $\alpha\beta$ =	3752,54	3864,53
add. $\beta\gamma$ =	3319,76	3418,84
erit $\alpha\gamma = ak$ =	7072,30	7283,37
Nunc a $l\,ck$ =	4,1241765	4,1294001
ſubtr. $l\,ak$ =	3,8495607	3,8623324
l tang. cak =	10,2746158	10,2670677
& cak =	62°, 0′,58″	61°,36′,4″
Ab $l\,ck$ =	4,1241765	4,1294001
ſubtr. l ſin cak =	9,9459999	9,9443135
erit $l\,ac$ =	4,1781766	4,1850866
& ac =	15072,20	15313,93
BS =	99576,55	99576,55
ſubtr. Bβ =	8400,00	8500,00
erit Sβ =	91176,55	91076,55
& l Sβ =	4,9598831	4,9594065
add. { l ſin Φ =	9,9799883	9,9807530
add. { l coſ. Φ =	9,4723364	9,4642513
erit l S Q =	4,9398714	4,9401595
$l\,\beta$ Q =	4,4322195	4,4236578
ergo β Q =	27053,26	26525,15
ſubtr. $\alpha\beta$ =	3752,54	3864,53
add. $\beta\gamma$ =	3319,76	3418,84
erit α Q =	23300,72	22660,62
γ Q =	30373,02	29943,99

Nunc

Nunc a l Q S =	4,9398714	4,9401595
ſubtr. (l Q α =	4,3673694	4,3552719
(l Q γ =	4,4824880	4,4763098
erit l tang. Q α S =	10,5725020	10,5848876
l tang. Q γ S =	10,4573834	10,4638497
Ergo ang. Q α S =	75°, 1′, 6″	75°,25′,16″
Q γ S =	70°,46′,11″	71°, 2′, 0″
Jam a l Q S =	4,9398714	4,9401595
ſubtr. (l ſin Q α S =	9,9849810	9,9857864
(l ſin Q γ S =	9,9750650	9,9757570
erit l S α =	4,9548904	4,9543731
l S γ =	4,9648064	4,9644025
A l a α =	4,5702326	4,5754624
ſubtr. l S α =	4,9548904	4,9543731
l tang. a S α =	9,6153422	9,6210893
ergo a S α =	22°,24′,45″	22°,40′,51″
a l a α =	4,5702326	4,5754624
ſubtr. l ſin a S α =	9,5812350	9,5861341
l S a =	4,9889976	4,9893283
ergo S a =	97498,42	97572,70
Porro a l c γ =	4,7031485	4,7083766
ſubtr. l S γ =	4,9648064	4,9644025
l tang. c S γ =	9,7383421	9,7439741
ergo c S γ =	28°,41′,54″	29°, 0′,45″
Nunc a l c γ =	4,7031485	4,7083766
ſubtr. l ſin c S γ =	9,6814203	9,6857422
l S c =	5,0217282	5,0226344
& S c =	105130,37	105349,97

§. XXXVIII.

Sit nunc primum orbita parabola, & quæratur tempus quod ad arcum ac percurrendum abſolvitur, uti § 32 eſt factum.

add. $f = Sa =$	97498,42	97572,70
add. $g = Sc =$	105130,37	105349,97
add. $k = ac =$	15072,20	15313,93
erit $f + g + k =$	217700,99	218236,60
& $\frac{f+g+k}{2} =$	108850,50	109118,30
ſubtr. $k =$	15072,20	15313,93
erit $\frac{f+g-k}{2} =$	93778,30	93804,37
Ad $l\frac{f+g+k}{2} =$	5,0368304	5,0378976
add. ſemiſſis =	2,5184152	2,5189488
	7,5552456	7,5568464
ſubtr. $l\,3\,m$	6,0621886	6,0621886
l part. prioris	1,4930570	1,4946578
Porro $l\frac{f+g-k}{2} =$	4,9721024	4,9722231
add. ſemiſſis =	2,4860512	2,4861116
	7,4581536	7,4583347
ſubtr. $l\,3\,m$	6,0621886	6,0621886
l part. poſterior. =	1,3959650	1,3961461
Ergo pars prior =	31,12125	31,23617
ſubtr. pars poſter. =	24,88657	24,89695
Tempus per ac in d.	6,23468	6,33922

Cum igitur tempus obſervatum ſit 6,30971. d. verus valor ipſius $B\beta$ manifeſto intra limites 8400 & 8500 continetur, & quia in tam parvo intervallo incrementa temporum

rum ſunt proxime uti incrementa ipſius $B\beta$; ſumamus differentias:

$$\frac{6,33922 - 6,23468}{0,10454} : 100 = \frac{6,30971 - 6,23468}{0,07503} : 71,77$$

erit ergo valor proxime verus ipſius $B\beta = 8471$; qui autem ex præcedentibus hypotheſibus exactius determinabitur interpolationibus in ſubſidium vocatis:

$B\beta = 8000 +$	0,	400,	500,	1000	
tempus per $ac =$	5,82218,	6,23468;	6,33922;	6,76979	
diff. I.		103, 10	104, 54	86, 11	
diff. II.			$\frac{1,44}{500}$	$- \frac{18,43}{600}$	
diff. III.				$- \frac{100,79}{3000000}$	

Si jam ponatur $B\beta = 8000 + x$, erit tempus per $ac = 5,82218 + x . 103,10 + x(x-400)\frac{1,44}{500} - x(x-400)(x-500)\frac{100,79}{3000000}$

Hinc ſi ſumatur $x = 470$, fit tempus per $ac = 6,30803$

ſin ſumatur $x = 471$, fit tempus per $ac = 6,30907$

ſi ſumatur $x = 472$, fit tempus per $ac = 6,31012$

fiat ergo ut $105 : 1 = 64 : 0,61$, ex quo verus valor ipſius $B\beta$ colligitur fore $= 8471,61$, qui non multum diſcrepat ab eo, quem duo valores poſteriores ſoli ſuppeditaverunt.

§. XXXIX.

Nunc in altera hypotheſi, qua orbita cometæ ſtatuitur ellipſis, in qua ſemilatus rectum eſt ad diſtantiam perihelii a ſole ut $2-\alpha$ ad 1, denotante $\alpha = \frac{1}{350}$, quæratur tempus ad ſpatium ac percurendum requiſitum modo §. 34. expoſito, ſeu adhibeatur hæc formula: $\sin \frac{1}{2} v = \frac{\sqrt{(k+B)(k+C)} - \sqrt{(k-B)(k-C)}}{2k}$)

 Cum

Cum ſit g =	105130,37	105349,97
& k =	15072,20	15313,93
erit $g - k$ =	90058,17	90036,04
& $g + k$ =	120202,57	120663,90
& $\frac{g + k}{2}$ =	60101,26	60331,95
tum vero eſt f =	97498,42	97572,70
ad $l\,\frac{g + k}{2}$ =	4,7788835	4,7805474
addatur $l\,(g - k)$ =	4,9545231	4,9544164
	9,7334066	9,7349638
ſubtr. $l\,f$ =	4,9889976	4,9893283
	4,7444090	4,7456355
ergo $\frac{(g-k)(g+k)}{2f}$ =	55514,84	55671,84
ſubtr. $\frac{1}{2} f$ =	48749,21	48786,35
erit B =	6765,63	6885,49
ob k =	15072,20	15313,93
erit k + B =	21837,83	22199,42
k − B =	8306,57	8428,44
Porro ab g =	105130,37	105349,97
ſubtr. f =	97498,42	97572,70
$g - f$ =	7631,95	7777,27
& C = $\frac{50}{49}(g-f)$ =	7787,70	7935,99
k =	15072,20	15313,93
erit k + C =	22859,90	23249,92
k − C =	7284,50	7377,94
add. $l\,(k+B)$ =	4,3392095	4,3463416
add. $l\,(k+C)$ =	4,3590743	4,3664215
	8,6982838	8,7127631
	4,3491419	4,3563815
ſubtr. $l\,2k$ =	4,4792067	4,4861167
	9,8699352	9,8702648

Ergo

Ergo $= \frac{\sqrt{(k+B)\ (k+C)}}{2\ k} =$	0,741199	0,741762
add. $\left\{ l\ (k-B) = \right.$	3,9194217	3,9257472
$l\ (k-C) =$	3,8623997	3,8679352
	7,7818214	7,7936824
	3,8909107	3,8968412
ſubtr. $l\ 2\ k =$	4,4792067	4,4861167
	9,4117040	9,4107245
ergo $\frac{\sqrt{(k-B)\ (k-C)}}{2\ k} =$	0,258050	0,257468
a $\frac{\sqrt{(k+B)\ (k+C)}}{2\ k} =$	0,741199	0,741762
ſin $\frac{1}{2}\ v =$	0,483149	0,484294
Hinc $\frac{1}{2}\ v =$	28°,53′,28½″	28°,57′,58½″
& $v =$	57 ,46 ,57	57 ,55 ,57
Quia eſt $f =$	97498,42	97572,70
erit $\frac{50}{99}\ f =$	49241,63	49279,14
$\frac{49}{99}\ f =$	48256,80	48293,56
$l\ \frac{49}{99}\ f =$	4,6835585	4,6838892
add. l coſ. $v =$	9,7268369	9,7250275
	4,4103954	4,4089167
ergo $\frac{49}{99}\ f$ coſ. $v =$	25727,38	25639,92
$\frac{50}{99}\ f =$	49241,63	49279,14
erit $a =$	74969,01	74919,06
& $l\ a =$	4,8748818	4,8745924
Cum ſit B $=$	6765,63	6885,49
addatur $f =$	97498,42	97572,70
erit B $+ f =$	104264,05	104458,19
l (B$+f$) $=$	5,0181345	5,0189425
ſubtr. $l\ g =$	5,0217282	5,0226344

l B$+f$

$l\ \frac{B+f}{g} = l$ coſ. q =	9,9964063	9,9963081
Ergo $q = a$Sc =	7°,21′,39″	7°,27′,37″
Ad $l\ a$ =	4,8748818	4,8745924
add. $l\ \sqrt{a}$ =	2,4374409	2,4372962
$l\ a\sqrt{a}$ =	7,3123227	7,3118886
add. $l\ \sqrt{\frac{100}{99}}$ =	0,0021824	0,0021824
	7,3145051	7,3140710
ſubtrahatur	5,5850673	5,5850673
l N =	1,7294378	1,7290037
Nunc eſt $\frac{1}{2}$ PSa =	28°,53′,28½″	28°,57′,58½″
Ergo $l\ t$ =	9,7418057	9,7431584
$l\ t^3$ =	9,2254171	9,2294752
$l\ t^5$ =	8,7090285	8,7157920
Hinc t =	0,551830	0,553549
t^3 =	0,168041	0,169619
t^5 =	0,051171	0,051975
Ad t =	0,551830	0,553549
add. $\frac{1}{3}\ t^3$ =	0,056014	0,056539
	0,607844	0,610089
ſubtr. $\frac{1}{3.50}\ t^3$ =	0,001120	0,001131
	0,606724	0,608958
ſubtr. $\frac{1}{5.50}\ t^5$ =	204	208
	0,606520	0,608750
ſubtr. $\frac{1}{6.50.50}\ t^3$ =	11	11
	0,606509	0,608739
add. $\frac{1}{20.50.50}\ t^5$ =	1	1
T =	0,606510	0,608740
ergo l T =	9,7828380	9,7844318
add. l N =	1,7294378	1,7290037

Tempus

	1,5122758	1,5134355
Tempus a P ad *a* =	32,52938	32,61636
Ad $\frac{1}{2}$ PS*a*	28°,53′,28½″	28°,57′,58½″
add. $\frac{1}{2}$ *a*S*c* =	3 ,40 ,49½	3 ,43 ,48½
erit $\frac{1}{2}$ PSc =	32 , 34, 18	32 , 41, 47
Ergo $l\ t$ =	9,8053854	9,8074672
$l\ t^3$ =	9,4161562	9,4224016
$l\ t^5$ =	9,0269270	9,0373360
hinc t =	0,6388302	0,6418997
t^3 =	0,2607091	0,2644854
t^5 =	0,1063964	0,1089772
Ad t =	0,6388302	0,6418997
add. $\frac{1}{3}\ t^3$ =	0,0869030	0,0881618
	0,7257332	0,7300615
ſubtr. $\frac{1}{3.50}\ t^3$ =	0,0017380	0,0017632
	0,7239952	0,7282983
ſubtr. $\frac{1}{5.50}\ t^5$ =	4256	4359
	0,7235696	0,7278624
ſubtr. $\frac{1}{6.50.50}\ t^3$ =	174	176
	0,7235522	0,7278448
add. $\frac{1}{20.50.50}\ t^5$ =	21	21
T =	0,7235543	0,7278469
l T =	9,8594711	9,8620400
add. l N =	1,7294378	1,7290037
	1,5889089	1,5910437
Tempus a P ad *c* =	38,80689	38,99813
Tempus a P ad *a* =	32,52938	32,61636
Tempus ab *a* ad *c* =	6,27751	6,38177

§. XL.

Horum duorum temporum itaque alterum majus eſt, alterum minus, quam obſervatum 6, 30971 dierum, & ſumtis differentiis hinc concluditur verum valorem rectæ $B\beta$ eſſe 8430,88. Quo autem hunc valorem accuratius inveniamus, interpolationem ex omnibus quatuor pertractatis valoribus pro $B\beta$ aſſumtis concludamus. Cum igitur poſito

$B\beta$	$= 8000 +$	0	$+ 400$	$+ 500$	$+ 1000$
ſit temp. per ac	$=$	5,85980;	6,27751,	6,38177;	6,81223
Diff. I.		104,43,	104,26;	86,09	
Diff. II.		−0,00034;	− 0,03030		
Diff. III.			− 0,00002996		

unde colligitur, ſi capiatur $B\beta = 8000 + x$, fore tempus per $ac = 5,85980 + x$, $104,43 - x\,(x - 400)$ $\cdot 0,00034 - x\cdot(x - 400)\,(x - 500)$, $0,00002996$ ſi ergo ſit $B\beta = 8430,00$, erit tempus per $ac = 6,30882$, ſin autem $B\beta = 8431,00$, erit tempus per $ac = 6,30986$ Ergo fiat $104 : 1 = 89 : 0,85$, ideoque habebitur $B\beta = 8430,85$, qui eſt verus valor ipſius $B\beta$, in hypotheſi orbitæ ellipticæ, cujus ſemilatus rectum eſt ad diſtantiam perihelii a ſole ut $2 - \frac{1}{50}$ ad 1 ſeu ut 99 ad 50.

§. XLI.

Inventis ergo pro utraque hypotheſi veris ipſius $B\beta$ valoribus, ipſam cometæ orbitam definiamus ita, ut inde ad datum quodvis tempus locus cometæ computari poſſit.

Eſt

	Pro Parabola	Pro ellipſi.
Eſt ergo $B\beta =$	8471,61	8430,85
ſubtr. $Bm =$	5973,15	5973,15
erit $\beta m =$	2498,46	2457,70
addatur $mn =$	1378,85	1378,85
& $\beta n =$	3877,31	3436,55
Tum ab $BS =$	99576,55	99576,55
ſubtr. $B\beta =$	8471,61	8430,85
erit $S\beta =$	91104,94	91145,70
Jam *a* $l\beta n =$	3,5885305	3,5839408
ſubtr. $l\beta m =$	3,3976724	3,3905289
$l\frac{\beta n}{\beta m} =$	0,1908581	0,1934119
add. $l\frac{m}{n} =$	0,0532181	0,0532181
	0,2440762	0,2466300
add. l cot. $AmB =$	9,9947234	9,9947234
	0,2387996	0,2413534
Ergo $\frac{m.\beta n}{n.\beta m} =$	1,754189	1,764534
& numerator	2,754189	2,764534
$\frac{m.\beta n}{n.\beta m}$ cot. $AmB =$	1,733004	1,743125
ſubtr. cot. $BnC =$	0,889734	0,889734
denominator	0,843270	0,853391
Ergo l numerator.	10,4399937	10,4416219
l denominat.	9,9259667	9,9311480
l tang. $\Phi =$	10,5140270	10,5104739
Ergo $B\beta\gamma = \Phi =$	72°,58′,36″	72°,50′,42″
$\alpha m \beta =$	45 ,20 ,53	45 ,20 ,53
$A\alpha\beta =$	27 ,37 ,43	27 ,29 ,49
$\beta n \gamma =$	48 ,20 ,22	48 ,20 ,22
$C\gamma\beta =$	58 ,41 , 2	58 ,48 ,56

Jam a $l\,\beta\,m$ =	3,3976724	3,3905289
ſubtr. l ſin $A\,\alpha\,\beta$ =	9,6662730	9,6643612
	3,7313994	3,7261677
add. { l ſin $\alpha\,m\,\beta$ =	9,8521072	9,8521072
add. { l ſin Φ =	9,9805422	9,9802354
$l\,\alpha\,\beta$ =	3,5835066	3,5782749
$l\,\alpha\,m$ =	3,7119416	3,7064031
ergo $\alpha\,\beta$ =	3832,71	3786,82
$\alpha\,m$ =	5151,59	5086,31
add. $A\,m$ =	8126,40	8126,40
ergo $A\,\alpha$ =	13277,99	13212,71
Porro a $l\,\beta\,n$ =	3,5885305	3,5839408
ſubtr. l ſin $C\,\gamma\,\beta$ =	9,9316168	9,9322225
	3,6569137	3,6517183
add. { l ſin $\beta\,n\,\gamma$ =	9,8733764	9,8733764
add. { l ſin Φ =	9,9805422	9,9802354
$l\,\beta\,\gamma$ =	3,5302901	3,5250947
$l\,\gamma\,n$ =	3,6374559	3,6319537
$\beta\,\gamma$ =	3390,70	3350,38
$\gamma\,n$ =	4339,66	4285,03
add. $C\,n$ =	6840,66	6840,66
erit $C\,\gamma$ =	11180,32	11125,63

§. XLII.

His inventis erit ut ſupra vidimus $a\alpha = A\alpha$ tang. latitudinis, & $c\gamma = C\gamma$ tang. latitud. atque hinc reperietur diſtantia cometæ a terra vera tempore primæ & ultimæ obſervationis: nempe $Aa = \frac{a\,\alpha}{ſin.\,latit.}$ & $Cc = \frac{c\,\gamma}{ſin.\,latit.}$. Tum vero ut ante longitudo cordæ ac definietur.

Erit

Erit nempe $l\,A\,\alpha$ =	4,1231325	4,1209919
add. l tang. lat. =	10,4508513	10,4508513
$l\,a\,\alpha$ =	4,5739838	4,5718432
ſubtr. l ſin. lat. =	9,9743466	9,9743466
$l\,A\,\alpha$ =	4,5996372	4,5974966
I. Obſ. diſt. Cometæ a Terra =	39777,5	39581,9
Porro $l\,C\,\gamma$ =	4,0484543	4,0463246
add. l tang. lat.	10,6584481	10,6584481
$l\,c\,\gamma$ =	4,7069024	4,7047727
ſubtr. l ſin lat. =	9,9897766	9,9897766
$l\,C\,c$ =	4,7171258	4,7149961
III. Obſer. diſt. Cometæ a terra =	52134,6	51889,5
Deinde eſt $c\gamma$ =	50921,66	50672,55
ſubtr. $a\,\alpha$ =	37495,90	37311,54
erit $c\,k$ =	13425,76	13361,01
ad $\alpha\,\beta$ =	3832,71	3786,82
add. $\beta\,\gamma$ =	3390,70	3350,38
$\alpha\gamma = a\,k$ =	7223,41	7137,20
A $l\,c\,k$ =	4,1279289	4,1258393
ſubtr. $l\,a\,k$ =	3,8587423	3,8535279
l tang. $c\,a\,k$ =	10,2691866	10,2723114
& ang. $c\,a\,k$ =	61°,43′, 5″	61°,53′,23″
jam a $l\,c\,k$ =	4,1279289	4,1258393
ſubtr. l ſin. $c\,a\,k$ =	9,9447918	9,9454895
$l\,a\,c$ =	4,1831371	4,1803498
& $a\,c$ =	15245,34	15147,80

§. XLIII.

Si jam corda $a\,c$ producatur, donec rectæ $\gamma\,\alpha$ continuatæ occurrat in N, erit N punctum in orbita cometæ ſimulque in ecliptica; quare ſi ducatur recta S N, erit hæc S N

 linea

linea nodorum. Cum autem ſit ang. $a N \alpha = c a k$, fiet $\frac{a \alpha}{N \alpha} =$ tang. $c a k$ ideoque $\alpha N = \frac{a \alpha}{tang. c a k}$. Demiſſa autem ex S in $\alpha\gamma$ productam normali SQ, recta $S\alpha$, & $S\gamma$ definientur ut ante, & ob angulos $BS\alpha$, $BS\gamma$ datos obtinebitur longitudo cometæ heliocentrica in prima & tertia obſervatione.

a $l\, a\alpha =$	4,5739838	4,5718432
ſubtr. l tang. $c a k =$	10,2691866	10,2723114
$l\, \alpha N =$	4,3047972	4,2995318
Ergo $\alpha N =$	20174,24	19931,12
Ad $l\, S\beta =$	4,9595419	4,9597361
add. l ſin $\Phi =$	9,9805422	9,9802354
add. l coſ. $\Phi =$	9,4665134	9,4697598
$l\, SQ =$	4,9400841	4,9399715
$l\, \beta Q =$	4,4250553	4,4294959
Ergo $\beta Q =$	26610,64	26884,13
ſubtr. $\alpha\beta =$	3832,71	3786,82
add. $\beta\gamma =$	3390,70	3350,38
Ergo $\alpha Q =$	22777,93	23097,31
$\gamma Q =$	30001,34	30234,51
A $l\, SQ =$	4,9400841	4,9399715
ſubtr. $l\, \alpha Q =$	4,3575143	4,3635614
ſubtr. $l\, \gamma Q =$	4,4771406	4,4805029
l tang. $S\alpha Q =$	10,5825698	10,5764101
l tang. $S\gamma Q =$	10,4629435	10,4594686
Ergo $S\alpha Q =$	75°,20′,48″	75°, 8′,47″
$S\gamma Q =$	70 ,59 , 48	70 ,51 , 18
A $l\, SQ =$	4,9400441	4,9399715
ſubtr. l ſin. $S\alpha Q =$	9,9856394	9,9852396
ſubtr. l ſin. $S\gamma Q =$	9,9756614	9,9752901

erit

erit $l\,S\alpha =$	4,9544447	4,9547319
$l\,S\gamma =$	4,9644227	4,9646814
$A\,Q\alpha S =$	75°,20′,48″	75°, 8′,47″
ſubtr. $Q\beta S =$	72 ,58 ,36	72 ,50 ,42
addatur $\alpha S\beta =$	2°,22′,12″	2°,18′, 5″
ad longit. B $=$	5^s,24°,24′,26″	5^s,24°,24′,26″
I.Obſ.Longitudo Com.heliocentr.	5^s,26 ,46′,38	5^s,26°,42′,31″
$A\,Q\beta S =$	72°,58′,36″	72°,50′,42″
ſubtr. $Q\gamma S =$	70 ,59 ,48	70 ,51 ,18
$\beta S\gamma =$	1°,58′,48″	1°,59′,26″
α long. B $=$	5^s,24°,24′,26″	5^s,24°,24′,26″
III.Obſ. Longit. Com. heliocentr.	5^s,22°,25′,38″	5^s,22°,25′, 0″
Porro a $l\,a\alpha =$	4,5739838	4,5718432
ſubtr. $l\,S\alpha =$	4,9544447	4,9547319
l tang. $aS\alpha =$	9,6195391	9,6171113
I.Obſ. latitudo Com. helioc. $aS\alpha =$	22°,36′,29″	22°,29′,41″
a $l\,a\alpha =$	4,5739838	4,5718432
ſubtr. l ſin $aS\alpha =$	9,5848118	9,5827431
$l\,Sa =$	4,9891720	4,9891001
I. Obſ. Diſt. Com. a Sole $Sa =$	97537,59	97521,45
Jam a $l\,c\gamma =$	4,7069024	4,7047727
ſubtr. $l\,S\gamma =$	4,9644227	4,9646814
l tang. $cS\gamma =$	9,7424797	9,7400913
III.Obſ.Lat.Com.heliocent. $cS\gamma =$	28°,55′,44″	28°,47′,44″
a $l\,c\gamma =$	4,7069024	4,7047727
ſubtr. l ſin. $cS\gamma =$	9,6845973	9,6827639
$l\,Sc =$	5,0223051	5,0220088
III. Obſ. diſt. Com. a Sole $Sc =$	105270,13	105198,32

§. XLIV.

Inventis his lineis erit $QN = \alpha Q - \alpha N$. Hinc erit tang $QSN = \frac{QN}{QS}$; ergo $\beta SN = \beta SQ - QSN =$ 90° —

$90^\circ - \Phi - QSN$, ſicque innoteſcit poſitio lineæ nodorum SN, atque longitudo nodi aſcendentis N heliocentrica. Tum in SN productam ex α demittatur perpendicularis αR, erit $\alpha R = S\alpha$ ſin αSR, eſtque $\alpha SR = 90^\circ - Q\alpha S - QSN$, & $SR = S\alpha$ coſ. αSR; hincque ducta aR, quæ erit normalis ad SR reperietur tang. $aR\alpha = \frac{a\alpha}{\alpha R}$; eritque ang. $aR\alpha$ = inclinationi orbitæ cometæ ad eclipticam. Tum vero eſt $aR = \frac{a\alpha}{\text{ſin. } aR\alpha}$ & ſin. $aSN = \frac{aR}{Sa}$. eritque ang. aSN diſtantia cometæ heliocentrica a nodo tempore primæ obſervationis. Denique angulus aSc hinc reperietur, quod ſit coſ. $aSc = \frac{Sa^2 + Sc^2 - ac^2}{2\,Sa.\ Sc}$, hincque coſ. $\frac{1}{2}\,aSc = \sqrt{\left(\frac{Sa+Sc+ac}{2\,Sa}\right)\left(\frac{Sa+Sc-ac}{2\,Sc}\right)}$.

Ab αQ =	22777,93	23097,31
ſubtr. αN =	20174,24	19931,12
erit QN =	2603,69	3166,19
a $l\,QN$ =	3,4155893	3,5005369
ſubtr. $l\,QS$ =	4,9400841	4,9399715
l tang. QSN =	8,4755052	8,5605654
QSN =	1°,42′,43″	2° 4′,55″
add. Φ =	72,58, 36	72,50, 42
	74,41, 19	74,55,37
Ergo βSN =	15°,18′,41″	15°, 4′, 23″
add. long. B =	5ſ,24°,24′,26″	5ſ,24°,24′,26″
Longitudo nodi aſcend.	6ſ,9°,43′, 7″	6ſ,9°,28′,49″
ad QSN addatur $Q\alpha S$ =	75°,20′,48″	75°, 8′, 47″
	77°, 3′, 31	77°,.13′, 42″
erit αSR =	12°,56′,29″	12°, 46′, 18″

ad

ad *l* S. α =	4,9544447	4,9547319
add. *l* ſin. α S R =	9,3501591	9,3445224
l α R =	4,3046038	4,2992543
a *l* *a* α =	4,5739838	4,5718432
l tang. *a* R α =	10,2693800	10,2725889
Inclinatio ad eclipt. *a* R α =	61°,43′,44″	61°,54′,18″
Ab *l* *a* α =	4,5739838	4,5718432
ſubtr. *l* ſin. *a* R α =	9,9448360	9,9455513
l *a* R =	4,6291478	4,6262919
ſubtr. *l* S *a* =	4,9891720	4,9891001
l ſin *a* S N =	9,6399758	9,6371918
I. Obſ. diſt. com. hel. a nod. *a*SN =	25°,52′,49″	25°,42′,10″
S *a* =	97537,59	97521,45
S *c* =	105270,13	105198,32
a *c* =	15245,34	15147,80
	218053,06	217867,57
$\frac{Sa + Sc + ac}{2}$ =	109026,53	108933,78
$\frac{Sa + Sc - ac}{2}$ =	93781,19	93785,98
ad *l* $\frac{Sa + Sc + ac}{2}$ =	5,0375321	5,0371625
add. *l* $\frac{Sa + Sc - ac}{2}$ =	4,9721158	4,9721379
	10,0096479	10,0093004
ſubtr. *l* S *a* =	4,9891720	4,9891001
	5,0204759	5,0202003
ſubtr. *l* S *c* =	5,0223051	5,0220088
	9,9981708	9,9981915
l coſ. ½ *a* S *c* =	9,9990854	9,9990957
Ergo ½ *a* S *c* =	3°,43′,1½″	3°,41′,46½″
& *a* S *c* =	7°,26′,3″	7°,23′,33″
III. Obſ. diſt. C. hel. a nod. *c*SN =	33°,18′,52″	33°,5′,43″

§. XLV.

§. XLV.

Fig. 2. Inventis nunc duabus cometæ a ſole diſtantiis $Sa = f$; $Sc = g$, cum corda $ac = k$ ſeu angulo $aSc = q$, orbita cometæ tam parabolica quam elliptica definiri poterit. Poſito enim angulo $PSa = v$, ſi fiat $B = g \text{ coſ. } q - f$ & $C = \frac{g-f}{1-\alpha}$, erit $\text{ſin } \tfrac{1}{2} v = \frac{\sqrt{(k+B)(k+C)} - \sqrt{(k-B)(k-C)}}{2k}$; ubi pro parabola eſt $\alpha = 0$, & pro ellipſi eſt $\alpha = \frac{1}{50}$. Tum invento angulo v erit $PS = a = \frac{f}{2-\alpha} + \frac{f(1-\alpha)}{2-\alpha} \text{ coſ. } v$.

$lSc = lg =$	5,0223051	5,0220088
add. l coſ. $aSc = l$ coſ. $q =$	9,9963340	9,9963751
$l\ g$ coſ. $q =$	5,0186391	5,0183839
ergo g coſ. $q =$	104385,25	104323,92
ſubtr. $Sa = f =$	97537,59	97521,45
erit $B =$	6847,66	6802,47
$ac = k =$	15245,34	15147,80
$k + B$	22093,00	21950,27
$k - B$	8397,68	8345,33
Ab $g =$	105270,13	105198,32
ſubtr. $f =$	97537,59	97521,45
$g - f =$	7732,54	7676,87
erit $C =$	7732,54	7833,54
$k =$	15245,34	15147,80
$k + C =$	22977,88	22981,34
$k - C =$	7512,80	7314,26
Ad $l(k+B) =$	4,3442547	4,3414399
add. $l(k+C) =$	4,3613099	4,3613754
	8,7055646	8,7028153

lV

$lV(k+B)(k+C) =$	4,3527823	4,3514077
ſubtr. $l2k =$	4,4841671	4,4813798
l part. prioris	9,8686152	9,8700279
Ad $l(k-B) =$	3,9241593	3,9214436
add. $l(k-C) =$	3,8758018	3,8641704
	7,7999611	7,7856140
$lV(k-B)(k-C) =$	3,8999806	3,8928070
ſubtr. $l\,2\,k =$	4,4841671	4,4813798
l part. poſt. $=$	9,4158135	9,4114272
Pars prior $=$	0,738950	0,741358
ſubtr. pars poſt. $=$	0,260503	0,257886
ſin $\frac{1}{2}v =$	0,478447	0,483472
Ergo $\frac{1}{2}v =$	28°,35′,2½″	28°,54′,44½″
& $aSP = v =$	57°,10′, 5″	57°,49′,29″
ſubtr. $aSN =$	25°,52,49	25 ,42,10
Diſt. perihelii a nodo $=$	31°,17′,16″	32°, 7′,19″
Cum ſit $f =$	97537,59	97521,45
erit $\frac{1-\alpha}{2-\alpha}f =$	48768,79	48358,19
Ad $l\frac{1-\alpha}{2-\alpha}f =$	4,6881420	4,6844700
add. l coſ. $v =$	9,7341409	9,7263287
	4,4222829	4,4107987
Ergo $\frac{(1-\alpha)f}{2-\alpha}$ coſ.$v =$	26441,31	25751,27
add. $\frac{f}{2-\alpha} =$	48768,79	49253,26
Ergo $PS = a =$	75210,10	75004,53
& $l\,a =$	4,8762762	4,8750875

§. XLVI.

Inventa orbita cometæ ſupereſt, ut tempus, quo cometa per perihelium tranſiit, definiamus, quod reperietur, ſi tempus determinemus, quo cometa a perihelio P ad locum a appulerit: Ponatur tang. $\frac{1}{2}$ a S P = tang. $\frac{1}{2}$ $v = t$, eritque tempus, quo cometa ex P ad a pervenit, in diebus expreſſum

$$= \frac{a \sqrt{a}}{384651,5 \sqrt{(1-\frac{1}{2}\alpha)}} \left(t + \frac{1}{3} t^3 - \frac{\alpha}{3} t^3 - \frac{\alpha}{5} t^5 - \frac{\alpha\alpha}{6} t^3 + \frac{\alpha\alpha}{20} t^5\right)$$

ſit vero $\frac{a \sqrt{a}}{384651,5 \sqrt{(1-\frac{1}{2}\alpha)}} = N$, & $T = t + \frac{1}{3} t^3 - \frac{\alpha}{3} t^3 - \frac{\alpha}{5} t^5 - \frac{\alpha\alpha}{6} t^3 + \frac{\alpha\alpha}{20} t^5)$ ut ſit tempus per Pa = N T.

Ad $l\, a$ =	4,8762762	4,8750875
add. $l \sqrt{a}$ =	2,4381381	2,4375437
$l\, a \sqrt{a}$ =	7,3144143	7,3126312
add. $l \frac{1}{\sqrt{(1-\frac{1}{2}\alpha)}}$ =	0,0000000	0,0021824
	7,3144143	7,3148136
ſubtr.	5,5850673	5,5850673
l N =	1,7293470	1,7297463
Jam eſt $l\, t$ =	9,7362818	9,7421838
$l\, t^3$ =	9,2088454	9,2265514
$l\, t^5$ =	— — —	8,7109190
t =	0,544856	0,552311
t^3 =	0,161750	0,168481
t^5 =	— — —	0,051394
Ad t =	0,544856	0,552311
add. $\frac{1}{3} t^3$ =	0,053917	0,056160
	0,598773	0,608471
ſubtr. $\frac{\alpha}{3}$ t^3 =	— — —	1123

ſubtr.

	— —	0,607348
ſubtr. $\frac{\alpha}{5} t^5$ =	— —	205
	— —	0,607143
ſubtr. $\frac{\alpha\alpha}{6} t^3$ =	— —	11
erit T =	0,598773	0,607133
add. *l* T =	9,7772622	9,7832838
add. *l* N =	1,7293470	1,7297463
	1,5066092	1,5130301
Tempus a perihelio ad *a* =	32,10771	32,58593
Cometa ergo a perihelio ad locum *a* pervenit — —	$32^d, 2^h, 35'$	$32^d, 14^h, 3'$
At cometa in *a* eſt verſatus Jan.	$59^d, 6, 49$	$59^d, 6^h, 49'$
Cometa in perihelio fuit M. Jan.	$27^d, 4^h, 14'$	$26^d, 16^h, 45'$

§. XLVII.

Invenimus ergo pro orbita cometæ duplicem figuram, alteram parabolam, alteram ellipſin, cujus ſemilatus rectum eſt ad diſtantiam perihelii a ſole ut 99 ad 50. Utraque autem per ſequentes conditiones determinatur.

Orbita Cometæ	Parabolica	Elliptica
1. Diſtantia Perihelii a ſole *a* =	75210,10	75004,53
& *l a* =	4,8762762	4,8750875
2. Semi-latus rectum *b* =	150420,20	148508,97
3. Cometa per perihelium tranſiit A. 1742. M. Jan.	$27^d, 4^h, 14'$	$26^d, 16^h, 45'$
4. Diſtantia heliocentrica perihelii a nodo =	$31°, 17', 16''$	$32°, 7', 19''$
5. Longitudo heliocentrica nodi aſcendentis =	$6^s, 9°, 43', 7''$	$6^s, 9°, 28', 49''$

6. Inclinatio orbitæ cometæ ad ad eclipticam =	61°,43′,44″	61°,54′,18″
Conſtantis numeri N logarithmus ſeu l N =	1,7293470	1,7297463

Atque ex his cognitis ad tempus quodvis datum locus cometæ poterit asſignari. Nam a tempore dato ſubtrahatur tempus, quo cometa in perihelio eſt verſatus, & reliquum in diei partibus decimalibus exprimatur, cujus numeri ſumatur logarithmus, a quo ſubtrahatur l N, & remanebit $l\,T = l(t + \frac{1}{3}t^3 - \frac{\alpha}{3}t^3 - \frac{\alpha}{5}t^5 - \frac{\alpha\alpha}{6}t^3 + \frac{\alpha\alpha}{20}t^5)$, unde valor anomaliæ veræ v per tabulam datam poterit inveniri, ſi pro ellipſi ea præcepta §. 11. tradita in ſubſidium vocentur. Tum ab anomalia vera v ſubtrahatur diſtantia perihelii a nodo, prodibitque diſtantia cometæ a nodo heliocentrica.

§. XLVIII.

Computemus ex utraque hypotheſi locum cometæ ad tempus obſeruationis ultimæ Mart. $17^d, 7^h, 50'$, quo cometæ longitudo eſt obſervata $2^s, 18°, 52'$, & latitudo borealis 63°, 13′. Pro hoc vero tempore colligitur locus terræ $= 6^s, 7°, 57', 30''$ & diſtantia terræ a ſole = 99970, 05. Jam tempus propoſitum ad menſem Januarium addendo 59^d, reductum erit.

Tempus propoſitum Jan. =	$76^d, 7^h, 50'$	$76^d, 7^h, 50'$
ſubtr. tempar. com. in perih. =	27 , 4 ,14	26 ,16 ,45
	$49^d, 3^h, 36'$	$49^d, 15^h, 5'$
Tempus a perihelio elapſum =	49 ,15000	49, 62847
l temp. =	1,6915235	1, 6957309
ſubtr. l N =	1,7293470	1, 7297463

erit

erit l T =	9,9621765	9,9659846
Anomalia vera u =	74°,56′29″	75°,18′,18″
$\frac{1}{2}$ u =	37°,28′,15″	37°,39′, 9″
l ſin $\frac{1}{2}$ u =	— — —	9,7859494
l (ſin $\frac{1}{2}$ u)^2 =	— — —	9,5718988
l tang. $\frac{1}{2}$ u =	— — —	9,8873722
	— — —	9,4592710
ſubtr. l $\frac{5}{2\alpha}$ =	— — —	2,0969100
	— — —	7,3623610
ſubtrahatur =	— — —	4,6855749
	— — —	2,6767861
erit ipſius Φ pars prior =	— — —	475,1″
l (ſin $\frac{1}{2}$ u)^2 =	— — —	9,5718988
add. l ſin u =	— — —	9,9855551
	— — —	9,5574539
ſubtr. l $\frac{15}{2\alpha}$ =	— — —	2,5740313
	— — —	6,9834226
ſubtrahatur	— — —	4,6855749
		2,2978477
erit ipſius Φ pars poſt.	— — —	198,5
Ergo Φ =	— — —	674″
ſeu Φ =	— — —	71′,14″
Anomalia vera v =	74°,56′,29″	75°,29′,32″
ſubtr. diſt. perih. a nodo =	31 ,17 ,16	32 , 7 ,19
Diſtantia cometæ a nodo =	43°,39′,13″	43°,22′,13″

§. XLIX.

Sit in triangulo ſphærico ☊ c γ latus ☊ γ ecliptica, & ☊ c orbita cometæ, erit angulus c ☊ γ inclinatio ad eclipticam. Tum ſumta c ☊ = diſtantiæ cometæ a nodo, erit c locus co- *Fig. IV.*

cometæ

metæ verus in orbita, unde ſi ad ☊ γ ducatur normalis $c\gamma$, erit arcus $c\gamma$ latitudo cometæ heliocentrica, & ſi ☊γ ſubtrahatur a longitudine nodi, prodibit cometæ longitudo heliocentrica. Erit vero ex natura triangulorum ſphæricorum ſin $c\gamma$ = ſin c☊, ſin c☊γ, & tang. ☊γ = coſ. c☊γ. tang. c☊. Quare erit.

☊ c	=	43°,39′,13″	43°,22°,13″
& c ☊ γ	=	61 ,43 ,44	61 ,54 ,18
Hinc l ſin c☊	=	9,8390359	9,8367736
add. l ſin c☊γ	=	9,9448359	9,9455489
l ſin $c\gamma$	=	9,7838718	9,7823225
Latitudo heliocentrica $c\gamma$	=	37°,26′,31″	37°,17′, 9″
Porro l tang. c☊	=	9,9795815	9,9752805
add. l coſ. c☊γ	=	9,6754522	9,6729609
l tang. ☊γ	=	9,6550337	9,6482414
ergo ☊γ	=	24°,19′, 4″	23°,59′,0″
ſubtr. a longitudine nodi		6ˢ,9°,43, 7	6ˢ,9°,28,49
Longitudo heliocentrica		5ˢ,15°,24′, 3″	5ˢ,15°,29′,49″

§. L.

Fig. V. Sit igitur S ſol & T locus terræ, tum extra planum ecliptica exiſtat cometa in c, unde in eclipticam cadat perpendiculum $c\gamma$, erit ducta Sγ, ang. $cS\gamma$ = latit. cometæ heliocentricæ: unde fit $c\gamma = Sc$ ſin $cS\gamma$ & $S\gamma = Sc$ coſ. $cS\gamma$ tum ex γ in CT demittatur perpendiculum γp, erit ob angulum γST datum, quippe qui eſt differentia inter longitudines terræ & cometæ heliocentricas, $\gamma p = S\gamma$ ſin γST & $Sp = S\gamma$ coſ. γST, hincque fit $Tp = ST - Sp$. Deinde invenitur tang. $ST\gamma = \frac{p\gamma}{pT}$, qui angulus ad longitudi-

gitudinem ſolis additus dabit cometæ longitudinem geocentricam. Denique erit $T\gamma = \frac{p\,\gamma}{\text{ſin } ST\gamma}$; & tang. $cT\gamma = \frac{c\gamma}{T\gamma}$; qui angulus $cT\gamma$ erit cometæ latitudo geocentrica, atque tandem ſit $\frac{c\gamma}{\text{ſin } cT\gamma} = Tc =$ diſtantiæ cometæ a terra. Oportet itaque primum diſtantiam cometæ a ſole Sc inveſtigari, quæ ex anomalia vera v definitur; erit nempe $Sc = \frac{a(2-\alpha)}{1+(1-\alpha)\text{coſ.}v}$, ubi pro parabola eſt $\alpha = 0$ & pro ellipſi $\alpha = \frac{1}{50}$.

Cum ſit $v =$	74°,56′,29″	75°,29′,32″
erit coſ.$v =$	259869	2505114
$1+(1-\alpha)\text{coſ.}v =$	1,2598069	1,2455012
$\frac{1+(1-\alpha)\text{coſ.}v}{2-\alpha} =$	0,6299035	0,6290410
hujus logarith. $=$	9,7992741	9,7986790
ſubtr. a $la =$	4,8762762	4,8750875
erit $l\,Sc =$	5,0770021	5,0764085
add. l ſin $cS\gamma =$	9,7838730	9,7823233
add. l coſ. $cS\gamma =$	9,8998039	9,9007075
$l\,c\gamma =$	4,8600751	4,8587318
$l\,S\gamma =$	4,9768060	4,9771160
A long. terræ	6ſ,7°,57′,30″	6ſ,7°,57′,30″
ſubtr. long. cometæ $=$	5,15,24, 3	5,15,29,49
erit $\gamma ST =$	22°,33′,27	22°,27′,41″
Ad $l\,S\gamma =$	4,9768060	4,9771160
add. l ſin $\gamma ST =$	9,5838903	9,5821324
add. l coſ.$\gamma ST =$	9,9654346	9,9657364
$l\,\gamma p =$	4,5606963	4,5592484
$l\,Sp =$	4,9422406	4,9428524

K. ergo

ergo $S\,p$ =	87546,86	87670,28
ſubtr. ab $S\,T$ =	99970,05	99970,05
erit $T\,p$ =	12423,19	12299,77
$a\;l\,p\,\gamma$ =	4,5606963	4,5592484
ſubtr. $l\,T\,p$ =	4,0942332	4,0898973
l tang. $S\,T\,\gamma$ =	10,4664631	10,4693511
Ergo $S\,T\,\gamma$ =	71°, 8′,21″	71°,15′,19″
ſeu $S\,T\,\gamma$ =	2ſ,11°,8′,21″	2ſ,11°,15′,19″
addatur longitudo ſolis	0, 7, 57, 30	0, 7 ,57,30
Longitudo cometæ geocentri.	2ſ, 19°, 5′,51″	2ſ,19°,12′,49″
$a\;l\,\gamma\,p$ =	4,5606963	4,5592484
ſubtr. l ſin $S\,T\,\gamma$ =	9,9760318	9,9763315
ſubtr. $l\,T\,\gamma$ =	4,5846645	4,5829169
$a\;l\,c\,\gamma$ =	4,8608751	4,8587318
l tang. $c\,T\,\gamma$ =	10,2762106	10,2758149
Latitudo geocentrica =	62°, 6′,10″	62°, 4′,52″

§. LI.

Longitudo cometæ inventa utraque non multum ab obſervata, quæ erat 2ſ, 18°, 52′ discrepat, at latitudo obſervata 63°, 13′ magis discrepat, quam ut error obſervationi tribui poſſit. Utroque autem caſu hypotheſis parabolica propius ad veritatem accedit, quam elliptica, ex quo ſequi videtur, cometæ orbitam non ſolum non eſſe ellipſin, ſed adeo hyperbolam, ita ut iſte cometa neque usquam ante apparuerit nobis, neque unquam ad nos ſit reverſurus. Atque hæc concluſio omnino firma ac veritati conſentanea eſſet cenſenda, ſi tam tres illæ obſervationes, ex quibus hæc theoria erat derivata, quam hæc quarta ſumma accuratione eſſent factæ, ut nulla prorſus correctione indigeant. Manifeſtum enim eſt ſi tres obſervationes

ad

ad theoriam condendam aſſumtæ aliquantillum immutentur, orbitam cometæ inde reſultantem ſenſibiliter discrepare, ſicque magis minusve reliquis obſervationibus convenire; ita ut ex discrepantia reliquarum obſervationum a calculo non tam ſpecies orbitæ cometæ, quam defectus obſervationum concludi poſſit. Interim tamen perſpicitur, ſi obſervationes hujus cometæ ad minuta prima eſſent exactæ, tum aberrationem orbitæ cometæ a figura parabolica cognoſci poſſe, dummodo orbitæ parabolicæ tanta latitudo concedatur, quanta ex minutis ſecundis, quæ in obſervationibus ſunt neglecta, oriri potuit. Applicatio enim orbitæ inventæ ad obſervationem ultimam in longitudine cometæ differentiam 7′, in latitudine vero 1′, 18″, produxit, qua differentiæ ortæ ſunt ab orbita parabolica & elliptica tam prope ad parabolicam accedente, ut ſemilatus rectum a dupla diſtantia verticis a foco tantum parte quinquageſima differret. At vero ut hæc concluſio ſit certa, requiritur ut orbita cometæ ex tribus obſervationibus eruta his ipſis obſervationibus exactiſſime reſpondeat, antequam eam cum reliquis obſervationibus comparare liceat. Cum igitur tam ob calculi prolixitatem, quam ob hypotheſin ratione arearum aliquantum a vero recedentem in ipſam orbitam inventam nonnulli errores irrepere potuerint, ab iis concluſio eruta primum purgari debebit, id quod methodo hactenus tradita minus exacte fieri poterit.

§. LII.

His igitur expoſitis quæſtio huc reducitur, ut, ſi orbita cometæ jam proxime fuerit cognita, ea corrigatur, atque ad veritatem penitus perducatur, cujus problematis utiliſſimi at-

que in theoria tam planetarum quam cometarum graviſſimi ſolutionem hic facilem & expeditam ſum traditurus. Pono igitur jam eſſe inventam orbitam parabolicam, quæ obſervationibus ſatis prope conveniat, talem enim methodus ante tradita facile largitur, atque multo minore labore quam hic impendi, ſi quidem concluſione proxime vera acquiescere velimus. Ex hac orbita parabolica igitur ſuperiori methodo inventa definiri debet orbita cometæ vera ſive ſit parabolica ſive elliptica, ſive hyperbolica, quæ ſex conditionibus determinabitur, primo ſcilicet diſtantia perihelii a ſole; ſecundo ratione hujus diſtantiæ ad ſemilatus rectum: tertio, tempore, quo cometa in perihelio eſt verſatus: quarto diſtantia perihelii a linea nodorum: quinto longitudine nodi ascendentis ſive descendentis heliocentrica; & ſexto inclinatione orbitæ cometæ ad eclipticam. Hæ autem ſex res orbitam cometæ veram determinantes non multum discrepabunt ab iis, quibus orbita cometæ prope vera parabolica determinatur, ita ut differentiæ futuræ ſint quam minimæ, atque inſtar differentialium tractari posſint. Sit igitur

In orbita cometæ.	Parabolica prope vera	Vera
1. Diſtantia perihelii a ſole =	a	$a + A$
2. Semilatus rectum ad diſtant. perih. a ſole =	2 : 1	2 − B : 1
3. Tempus quo cometa in perihelio fuit =	t	$t + T^{b}$
4. Diſtantia perihelii a nodo —	p	$p + P^{\circ}$
5. Longitudo nodi accendentis —	q	$q + Q^{\circ}$
6. Inclinatio orbitæ ad ecliptic. =	r	$r + R^{\circ}$

Hic igitur datæ ſunt quantit. a, t, p, q, & r, quippe quæ

ex

ex orbita cometæ parabolica priori methodo inventa ſunt cognitæ: litteræ autem majuſculæ A, B, T, P, Q & R denotant quantitates minimas incognitas, quarum valores ſi fuerint reperti, vera cometæ orbita erit cognita.

§. LIII.

Ad has autem incognitas A, B, T, P, Q & R inveniendas fingo ſex hypotheſes pro orbita cometæ, quarum unaquæque unico tantum capite ab orbita parabolica diſcrepet: atque hoc diſcrimen pro lubitu aſſumo datum & valde parvum, ſcilicet pro cometa, de quo eſt hic quæſtio, has ſex hypotheſes ita commodiſſime mihi quidem fingere videor.

Orbita cometæ	Parab.	Hyp. I.	Hyp. II.	Hyp. III.	Hyp. IV.	Hyp. V.	Hyp. VI.
1 Diſtantia perihelii a ſole =	a	$a+200$	a	a	a	a	a
2. Semi latus rectum ad diſt. perihelii a ſole. =	2 : 1	2 : 1	99 : 50	2 : 1	2 : 1	2 : 1	2 : 1
3. Tempus quo cometa in perihelio eſt verſatus =	t	t	t	$t+12^h$	t	t	t
4. Diſtantia perihelii a nodo heliocentr. =	p	p	p	p	$p+1°$	p	p
5. Longitudo nodi aſcend. =	q	q	q	q	q	$q+1°$	q
6. Inclinatio orbitæ ad eclipticam. =	r	r	r	r	r	r	$r+1°$

 Com-

Computetur nunc tam ex orbita parabolica, quam ex unaquaque hypotheſi locus cometæ ad tempus aliquod datum, quo obſervatio eſt inſtituta; ſitque longitudo cometæ ex orbita parabolica inventa $= m$; & latitudo $= n$, ex hypotheſibus vero prima, ſecunda, tertia, quarta, quinta & ſexta ortæ ſint cometæ pro eodem tempore longitudines m; $m^{I}, m^{II}, m^{III}, m^{IV}, m^{V}, m^{VI}$ & latitudines $n^{I}, n^{II}, n^{III}, n^{IV}, n^{V}, n^{VI}$. In hypotheſi igitur prima incrementum 200 ipſi a adjectum longitudinem auxit parte $m' - m$: & latitudinem parte $n' - n$; unde concluditur incrementum incognitum A longitudinem m aucturum eſſe parte $\frac{(m'-m)A}{200}$, & latitudinem n parte $\frac{(n'-n)A}{200}$, ſimili modo in ſecunda hyp. cum $\frac{1}{50}$ a ratione 2 : 1 demta augeat longitudinem m parte $m''-m$ & latitudinem parte $n''-n$, concluditur a parte incognita B, longitudinem augeri parte $(m''-m)\,50\,B$ & latitudinem parte $(n''-n)\,50\,B$. Atque pari modo reliquas hypotheſes perpendendo concluditur ex theoria cometæ vera, etſi incognita, prodituram fuiſſe cometæ longitudinem $= m + \frac{(m'-m)A}{200} + (m''-m)\,50\,B + \frac{(m^{III}-m)T}{12} + (m^{IV}-m)\,P + (m^{V}-m)\,Q + (m^{VI}-m)\,R$ & latitudinem $= n + \frac{(n'-n)A}{200} + (n''-n)\,50\,B + \frac{(n'''-n)\;T}{12} + (n^{IV}-n)\,P + (n^{V}-n)\,Q + (n^{VI}-n)\,R$. Cum igitur hæc

hæc longitudo & latitudo cum loco cometæ obſervato congruere debeat, ſi ponatur cometæ longitudo obſervata $= M$, & latitudo obſervata $= N$, habebuntur ſequentes duæ æquationes:

$$M - m = \frac{(m' - m)A}{200} + (m'' - m)\, 50\, B + \frac{(m''' - n)T}{12} + (m^{IV} - m)\, P + (m^{V} - m)\, Q + (m^{VI} - m)\, R;$$

atque

$$N - n = \frac{(n' - n)A}{200} + (n'' - n)\, 50\, B + \frac{(n''' - n)T}{12} + (n^{IV} - n)\, P + (n^{V} - n)\, Q + (n^{VI} - n)\, R,$$

in quibus brevitatis ergo ponatur $\frac{A}{200} = F$, $50\, B = G$ & $\frac{T}{12} = H$.

§. LIV.

Tres itaque obſervationes hoc modo pertractatæ ſex præbebunt hujusmodi æquationes, quæ ſufficere videantur ad ſex incognitas A, B, T, P, Q & R determinandas, verum quia orbita cometæ eſt planum per ſolem tranſiens, duæ latitudines tertiam jam determinant, ita ut una ſex illarum æquationum nihil ſit profutura. Hanc obrem ad iſtas ſex incognitas definiendas oportebit quatuor obſervationes adhiberi, quæ cum octo ſuppeditent æquationes, duæ ex latitudinibus ortæ omittantur, & ex ſex reliquis correctiones neceſſariæ determinentur. Quod ſi fuerit factum, orbita cometæ habebitur quatuor locis obſervatis ſatisfaciens, hincque omnibus numeris abſoluta, quæ ad quodvis tempus cometæ locum exactiſſime ſit monſtratura, ſiquidem obſervationes, quibus ad hoc inſtitutum uſi fuerimus fuerint ſatis exactæ, neque plus uno minuto primo

a veri-

a veritate aberrent. Quanquam autem obſervationes cometæ a Celeb. Delislio mecum communicatæ omnes vix ad hunc perfectionis gradum perductæ videntur, propterea quod Vir Celeberrimus mihi ſcribit, ſe nondum potuiſſe cunctas obſervationes pari diligentia examini ſubjicere, atque ab omnibus nævis purgare, tamen ex iis quatuor eligam ad methodum hanc illuſtrandam, quæ ſi fuerint ſatis accuratæ, orbitam cometæ veram indicabunt, ſin autem minus fuerint exactæ, laboris ſuſcepti tamen non pœnitebit, quod hoc modo ſaltem via ſit patefacta, per quam ſi obſervationes magis idoneæ impetrentur, ad ſcopum propoſitum pertingere liceat. Eligantur itaque quatuor ſequentes obſervationes.

Temp. medio	Long. com.	Lat. com.	Locus terræ	diſt ter-a ſol.
Febr. $25^d, 10^h, 0'$	$9^s, 24°, 13'$	$56°, 29'$	$5^s, 18°, 13', 5''$	99403, 70
Mart. $1^d, 15^h, 10'$	10, 22, 13	$75°, 29'$	5, 22, 25, 4	99520, 24
— $6^d, 14^h, 15'$	1, 12, 44	$77°, 37'$	5, 27, 20, 38	99660, 46
— $15^d, 6^h, 45'$	2, 16, 18	$65°, 20'$	6, 5, 56, 27	99910, 76

§. LV.

Sumamus jam orbitam cometæ parabolicam ante inventam, quam corrigi oporteat, eique adjungamus ſex hypotheſes ante deſcriptas.

Orbita Cometæ		Parabolica	Hypoth. I.	Hypoth. II.
I. Diſt. per. a ☉. a	=	75210	75410	75210
$l\ a$	=	4, 8762756	4, 8774289	4, 8762756
II. Ratio $b : a$	=	2 : 1	2 : 1	99 : 50
III. Temp. Per. t	=	Jan. $27^d, 4^h, 14'$	Jan. $27^d, 4^h, 14'$	Jan. $27^d, 4^h, 14'$
IV. Diſt. per a ☊ p	=	$31°, 17'$	$31°, 17'$	$31°, 17'$
V. Long. ☊ q	=	$6^s, 9°, 43'$	$6^s, 9°, 43'$	$6^s, 9°, 43'$
VI. Inclinatio r	=	$61°, 44'$	$61°, 44'$	$61°, 44$
& l N	=	1, 7293461	1, 7310741	1, 7315285

Hypoth.

Hypoth. III.	Hypoth. IV.	Hypoth. V.	Hypoth. VI.
$a =$ 75210	75210	75210	75210
$la =$ 4,8762756	4, 8762756	4, 8762756	4, 8762756
$b : a =$ 2 : 1	2 : 1	2 : 1	2 : 1
$t =$ Jan. $27^d, 16^h, 14$	Jan. $27^d, 4^h, 14'$	Jan. $27^d, 4^h, 14'$	Jan. $27^d, 4^h, 14'$
$p =$ $31^\circ, 17'$	$32^\circ, 17'$	$31^\circ, 17'$	$31^\circ, 17'$
$q =$ $6^s, 9^\circ, 43'$	$6^s, 9^\circ, 43'$	$6^s, 10^\circ, 43'$	$6^s, 9^\circ, 43'$
$r =$ $61^\circ, 44'$	$61^\circ, 44'$	$61^\circ, 44'$	$62^\circ, 44'$
$lN =$ 1,7293461	1, 7293461	1, 7293461	1, 7293461

Quod ſi ergo per 4 obſervationes modo indicato ſex correctiones A, B, T, P, Q & R definiantur ex totidem æquationibus, tum habebitur orbita cometæ vera quæ per ſex conditiones ſequentes determinabitur.

Orbita cometæ		Vera
I. Diſtantia perihelii a ſole	=	75210 + A
II. Semilatus rectum ad diſtantiam hanc		2 − B : 1
III. Tempus quo cometa fuit in perihel.		Jan. $27^d, 4^h, 14' + T^h$.
IV. Diſtantia perihelii a nodo ☊	=	$31^\circ, 17' + P^\circ$
V. Longitudo nodi aſcendentis ☊	=	$6^s, 9^\circ, 43' + Q^\circ$
VI. Inclinatio Orbitæ com. ad eclipt.		$61^\circ, 44' + R^\circ$.

ubi manifeſtum, eſt ſi valor ipſius B reperiatur affirmativus, fore orbitam cometæ ellipſin, ſin autem ſit B negativum, erit orbita cometæ hyperbola.

§. LVI.

Si jam ad tempus primæ obſervationis Febr. $25^d, 10^h, 0'$, tam ex orbita parabolica, quam ex ſex hypotheſibus computentur cometæ loca geocentrica, reperietur.

Cometæ	Longitudo	Latitudo
in Orbita Parabolica m	$9^s, 24^\circ, 31', 0''$	$n = 56^\circ, 55', 20''$
Hypoth. I. $m' =$	9, 23, 53, 0	$n' = 56, 43, 50$
Hypoth. II. $m'' =$	9, 25, 1, 30	$n'' = 56, 24, 50$
Hypoth. III. $m''' =$	9, 24, 22, 30	$n''' = 55, 12, 0$
Hypoth. IV. $m^{IV} =$	9, 22, 21, 50	$n^{IV} = 55, 2, 20$
Hypoth. V. $m^{V} =$	9, 22, 44, 10	$n^{V} = 55, 8, 40$
Hypoth. VI. $m^{VI} =$	9, 24, 7, 50	$n^{IV} = 56, 29, 20$
Per Obſerv. M.	9, 24, 13, 0	$N = 56, 29, 0$

Ex his itaque formentur æquationes modo (53) præſcripto

	in denis″		in denis″
$M - m = -18', 0''$	$-$ 108	$N - n = -26', 20''$	$-$ 158
$m' - m = -38, 0$	$-$ 228	$n' - n = -11, 30$	$-$ 69
$m'' - m = +30, 30$	$+$ 183	$n'' - n = -30, 30$	$-$ 183
$m''' - m = -8, 30$	$-$ 51	$n''' - n = -43, 20$	$-$ 260
$m^{IV} - m = -2^\circ, 9, 10$	$-$ 775	$n^{IV} - n = -1^\circ, 53, 00$	$-$ 678
$m^{V} - m = -1, 46, 50$	$-$ 641	$n^{V} - n = -1^\circ, 46, 40$	$-$ 640
$m^{VI} - m = -23, 10$	$-$ 139	$n^{VI} - n - - 26, 0$	$-$ 156

Fietque ex longitudinibus

$$-108 = -228F + 183G - 51H - 775P - 641Q - 139R.$$

& ex latitudinibus

$$-158 = -69F - 183G - 260H - 678P - 640Q - 156R.$$

§. LVII.

Ad tempus ſecundæ obſervationis Marti. $1^d, 15^h, 10'$ loca cometæ computata erunt.

Cometæ

Cometæ		Longitudo	Latitudo
Orbita Parab.	$m =$	10,21,54,50	$n = 75°,17',10''$
Hypoth. I.	$m' =$	10,20,41, 0	$n' = 75\ ,19,30$
Hypoth. II.	$m'' =$	10,22,23,40	$n'' = 74\ ,44,20$
Hypoth. III.	$m''' =$	10,20, 5,10	$n''' = 74,16,0$
Hypoth. IV.	$m^{IV} =$	10,21,50,40	$n^{IV} = 72,42,0$
Hypoth. V.	$m^{V} =$	10,15,27,10	$n^{V} = 74,0,40$
Hypoth. VI.	$m^{VI} =$	10,19,27,50	$n^{VI} = 74,42,50$
Per Obſerv.	$M =$	10,22,13, 0	$N = 75,29,0$

Ex his ergo fiet

	in denis''		in denis''
$M - m = +\ 18',10''$	$+\ 109$	$N - n = +\ 11',50''$	$+\ 71$
$m' - m = -\ 1°,13',50''$	$-\ 443$	$n' - n = +\ 2,20$	$+\ 14$
$m'' - m = +\ 0,28,50$	$+\ 173$	$n'' - n = -\ 32,50$	$-\ 197$
$m''' - m = -\ 1,49,40$	$-\ 658$	$n''' - n = -\ 61,10$	$-\ 367$
$m^{IV} - m = -\ 4,10$	$-\ 25$	$n^{IV} - n = -2°,35,10$	$-\ 931$
$m^{V} - m = -\ 6,27,40$	-2326	$n^{V} - n = -\ 1°,16,20$	$-\ 458$
$m^{VI} - m = -\ 2,27,0$	$-\ 882$	$n^{VI} - n = -\ 34,20$	$-\ 206$

Quare ex longitudinibus naſcitur hæc æquatio

$$+109 = -443F + 173G - 658H - 25P - 2326Q - 882R.$$

& ex latitudinibus hæc

$$+71 = +14F - 197G - 367H - 931P - 458Q - 206R.$$

§. LVIII.

Loca cometæ ad tertiæ obſervationis tempus quod erat Mart. 6^d, 14^h, $15'$, ſupputata ſunt.

Cometæ	Longitudo	Latitudo
Orb. Parabol. $m =$	$1^s, 12^\circ, 28', 30''$	$n = 77^\circ, 39', 10''$
Hypoth. I. $m' =$	1, 12, 43, 40	$n' =$ 77, 53, 10
Hypoth. II. $m'' =$	1, 10, 40, 0	$n'' =$ 77, 21, 30
Hypoth. III. $m''' =$	1, 10, 1, 10	$n''' =$ 77, 44, 30
Hypoth. IV. $m^{IV} =$	1, 12, 1, 20	$n^{IV} =$ 78, 41, 10
Hypoth. V. $m^{V} =$	1, 6, 28, 20	$n^{V} =$ 78, 54, 50
Hypoth. VI. $m^{VI} =$	1, 8, 25, 40	$n^{VI} =$ 78, 15, 0
Per Obſerv. $M =$	1, 12, 44, 0	$N =$ 77, 37, 0

Hinc ergo habetur.

$M - m = + 15', 30'', = + 93$	$N - n = - 2', 10 = - 13$
$m' - m = + 15, 10, = + 91$	$n' - n = + 14, 0 = + 84$
$m'' - m = - 1^\circ, 48, 30, = - 651$	$n'' - n = - 17, 40 = - 106$
$m''' - m = - 2^\circ, 27, 20, = - 884$	$n''' - n = + 5, 20 = + 32$
$m^{IV} - m = - 27, 10, = - 163$	$n^{IV} - n = + 62, 0 = + 372$
$m^{V} - m = - 6, 0, 10 = - 2161$	$n^{V} - n = + 75, 40 = + 454$
$m^{VI} - m = - 4, 2, 50 = - 1457$	$n^{VI} - n = + 95, 50 = + 575$

Longitudines ergo præbent hanc æquationem

$$+93 = +91F - 651G - 884H - 163P - 2161Q - 1457R.$$

Latitudines vero dabunt hanc

$$-13 = +84F - 106G + 32H + 372P + 454Q + 575R$$

§. LIX.

Ad quartam denique obſervationem, quæ facta eſt M. Mart. 15d, 6h, 45', loca cometæ ita ſe habent.

Cometæ

Cometæ	Longitudo	Latitudo
In Orb. Parabol. $m =$	$2^s, 16^\circ, 44', 10''$	$n = 64^\circ, 21', 50''$
Hypoth. I. $m' =$	2, 17, 5, 50	$n' =$ 64, 26, 10
Hypoth. II. $m'' =$	2, 15, 48, 40	$n'' =$ 64, 14, 30
Hypoth. III. $m''' =$	2, 16, 31, 10	$n''' =$ 64, 24, 0
Hypoth. IV. $\overset{IV}{m} =$	2, 18, 27, 40	$\overset{IV}{n} =$ 64, 44, 0
Hypoth. V. $\overset{V}{m} =$	2, 16, 42, 20	$\overset{V}{n} =$ 65, 29, 30
Hypoth. VI. $\overset{VI}{m} =$	2, 15, 53, 20	$\overset{VI}{n} =$ 65, 19, 20
Per Obſerv. M. $=$	2, 16, 18, 0	N $=$ 65, 20, 0

Quare erit

	in denis''		
$M - m = -26', 10''$	$-$ 157	$N - n = + 58', 10''$	$+$ 349
$m' - m = +21, 40$	$+$ 130	$n' - n = +$ 4, 20	$+$ 26
$m'' - m = -55, 30$	$-$ 333	$n'' - n = -$ 7, 20	$-$ 44
$m''' - m = -13, 0$	$-$ 78	$n''' - n = +$ 2, 10	$+$ 13
$\overset{IV}{m} - m = +1^\circ, 43, 30$	$+$ 621	$\overset{IV}{n} - n = +$ 22, 10	$+$ 133
$\overset{V}{m} - m = -$ 1, 50	$-$ 11	$\overset{V}{n} - n = +$ 67, 40	$+$ 406
$\overset{VI}{m} - m = -$ 50, 50	$-$ 305	$\overset{VI}{n} - n = +$ 57, 30	$+$ 345

Longitudines ergo præbent hanc æquationem

$$-157 = +130F - 333G - 78H + 621P - 11Q - 305R$$

& latitudines hanc

$$+349 = +26F - 44G + 13H + 133P + 406Q + 345R$$

§. LX.

Nacti ergo ſumus octo æquationes, quarum ſi duæ ex latitudinibus binarum mediarum obſervationum ortæ omittantur,

tantur, ſupererunt ſex ſequentes æquationes, ex quibus incognitas F, G, H, P, Q & R definiri oportebit.

$0 = +108 - 228F + 183G - 51H - 775P - 641Q - 139R$
$0 = +109 + 443F - 173G + 658H + 25P + 2326Q + 882R$
$0 = +93 - 91F + 651G + 884H + 163P + 2161Q + 1457R$
$0 = +157 + 130F - 333G - 78H + 621P - 11Q - 305R$
$0 = +158 - 69F - 183G - 260H - 678P - 640Q - 156R$
$0 = +349 - 26F + 44G - 13H - 133P - 406Q - 345R$

Unde ſex ſequentes valores reperiuntur pro R.

$R = -10,77697 - 1,64025F + 1,31655G - 0,36690H - 5,57554P - 4,61151Q$
$R = -0,12358 - 0,50227F + 0,19614G - 0,74603H - 0,02834P - 2,63719Q$
$R = -0,06383 + 0,06246F - 0,44681G - 0,60673H - 0,11187P - 1,48318Q$
$R = +0,51475 + 0,42623F - 1,09180G - 0,25574H + 2,03607P - 0,03606Q$
$R = +1,01282 - 0,44231F - 1,17308G - 1,66667H - 4,34615P - 4,10256Q$
$R = +1,01159 - 0,07536F + 0,12753G - 0,03768H - 0,38551P - 1,17681Q$

Subtrahantur ſinguli valores a primo & prodibunt quinque ſequentes æquationes.

$0 = +90055 - 113798F + 112041G + 37913H - 554720P - 197432Q$
$0 = +84080 - 170271F + 176336G + 23983H - 546367P - 312833Q$
$0 = +26222 - 206648F + 240835G - 1116H - 761161P - 457545Q$
$0 = -23585 - 119794F + 248963G + 129977H - 122939P - 50895Q$
$0 = -23462 - 156489F + 118902G - 32922H - 519003P - 343470Q$

Hinc reperiuntur quinque valores pro F.

$F = +0,79136 + 0,98456G + 0,33316H - 4,87460P - 1,73493Q$
$F = +0,49380 + 1,03562G + 0,14085H - 3,20881P - 1,83726Q$
$F = +0,12689 + 1,16544G - 0,05379H - 3,68337P - 2,21413Q$
$F = -0,19688 + 2,07826G + 1,08505H - 1,02625P - 0,42485Q$
$F = -0,14992 + 0,75981G - 0,21038H - 3,31655P - 2,19485Q$

Subtra-

Subtrahatur valor penultimus a reliquis orienturque, quatuor æquationes ſequentes.

$o = + 98824 - 109370\,G - 75189\,H - 384835\,P - 131008\,Q$

$o = + 69068 - 104264\,G - 94420\,H - 218256\,P - 141241\,Q$

$o = + 32377 - 91282\,G - 113884\,H - 265712\,P - 178928\,Q$

$o = + 4696 - 131845\,G - 129543\,H - 229030\,P - 177000\,Q$

Hinc quatuor obtinentur valores pro P

$P = + 0{,}25679 - 0{,}28420\,G - 0{,}19538\,H - 0{,}34043\,Q$

$P = + 0{,}31645 - 0{,}47771\,G - 0{,}43261\,H - 0{,}64713\,Q$

$P = + 0{,}12185 - 0{,}34354\,G - 0{,}42860\,H - 0{,}67338\,Q$

$P = + 0{,}02050 - 0{,}57567\,G - 0{,}56562\,H - 0{,}77283\,Q$

ſubtrahatur primus a reliquis, eritque

$o = + 5966 - 19351\,G - 23723\,H - 30670\,Q$

$o = - 13494 - 5934\,G - 23322\,H - 33295\,Q$

$o = - 23629 - 29147\,G - 37024\,H - 43240\,Q$

quae aequationes tres valores praebent pro Q

$Q = + 0{,}19452 - 0{,}63094\,G - 0{,}77349\,H$

$Q = - 0{,}40528 - 0{,}17822\,G - 0{,}70046\,H$

$Q = - 0{,}54646 - 0{,}67407\,G - 0{,}85624\,H$

ſubtrahatur media a reliquis erit

$o = + 59980 - 45272\,G - 7303\,H$

$o = - 14118 - 49585\,G - 15578\,H$

& reperientur pro G duo valores

$G = + 1{,}32488 - 0{,}16131\,H$

$G = - 0{,}28472 - 0{,}31416\,H$

qui a ſe invicem ſubducti relinquunt

$o = 160960 + 15285\,H$, ex qua reperitur

$H = - 10{,}53058$ & $T = - 126{,}36696$

Hinc

Hinc regrediendo erit

G = + 3, 02358 ; & B = + 0, 06047
Q = + 6, 43210
P = − 0, 73470
F = − 7, 21821 & A = − 1443, 642
R = − 5, 14819

§. LXI.

Orbita ergo cometæ, quantum quidem cum obſervationum bonitas, tum calculi non parum moleſti rectitudo concluſionem formare permittit, ſequentibus ſex conditionibus determinabitur.

1. Diſtantia perihelii a ſole a = 73766,36
   ejusque $l\,a$ = 4,8678584
2. Semi-latus rectum eſt ad a ut 193952 ad 100000
   ſeu ſi ratio hæc ponatur $2-\alpha$ ad 1 erit
   α = 0,06047
3. Cometa per perihelium tranſiit. M. Jan. $21^d, 21^h, 52'$
4. Diſtantia perihelii a nodo aſcend. $30°, 32', 55''$
5. Longitudo nodi aſcendentis : $6^s, 16°, 8', 55''$
6. Inclinatio orbitæ ad eclipticam $56°, 35', 7''$

Erit ergo orbita cometæ ellipſis, quia latus rectum minus eſt quam diſtantia perihelii a ſole quater ſumta; huiusque ellipſis ſemiaxis transverſus eſt $= \frac{a}{\alpha}$ = 1219883, poſito orbitæ terræ ſemiaxe transverſo = 100000. Cum igitur tempora periodica teneant axium transverſorum rationem ſesquiplicatam, erit tempus periodicum hujus cometæ circiter 42 annorum, & iſte cometa in aphelio ſuo duplo longius & ultra

a ſole

a ſole recedit, quam ſaturnus; Huic autem concluſioni non tuto confidere licet, priusquam cum de obſervationum bonitate fuerimus certiores, tum etiam univerſus calculus a principio fuerit repetitus. Quodſi autem hæc orbita cometæ veritati conſentanea eſſe comperiatur, tum ea in locum orbitæ denuo corrigendæ ſubſtituatur, atque ex quatuor obſervationibus ea per methodum modo expoſitam ad ſummum certitudinis gradum evehetur, quem laborem aliis ſuſcipiendum relinquo, methodum certam in hac diſſertatione tradidiſſe contentus.

§. LXII.

Antequam autem hic poſterior calculus pro correctionibus inſtitutus a capite ad calcem ſit repetitus & ab omni errore purgatus, expediet ex orbita cometæ parabolica præcipua phænomena perpendere ac dijudicare: ac primo quidem cometa terræ proximus fuiſſe deprehendetur Febr. d. 24. quo tempore ejus a terra diſtantia vix ſuperabat tertiam partem diſtantiæ terræ à ſole. Ante & poſt hoc tempus cometa longius a terra fuit remotus: & ſub finem apparitionis nondum ad ſolis diſtantiam receſſit; ita ut hic cometa a nobis conſpici nequeat, niſi ſole fuerit propior. Quare evenire utique poterit, ut hic cometa idem ſæpius ad perihelium revertatur, neque tamen nobis fiat conſpicuus: obſervari enim non poterit, niſi terra circa alterum cometæ nodum verſetur eo tempore quo cometa eclipticam in hac regione trajicit. Diameter apparens hujus cometæ d. 3. Mart. deprehenſa eſt circiter 25″, ex qua cum ejus diſtantia a terra illo tempore collata ſequitur

 diameter

diameter vera corporis ſeu nuclei cometæ vix ſemi diametrum terræ ſuperaſſe: erat ergo hic cometa Marte non multo minor, qui planeta cum in multo majoribus diſtantiis diſtincte conſpiciatur, veriſimile eſt cometas ob comam atque caudam plurimum offuscari. In denſa ſcilicet athmoſphæra radii ſolis non ſolum ingentem patiuntur refractionem, ſed etiam reflexio maxime impedietur; ita ut parvus radiorum numerus ad nos revertatur. Quam ob cauſam ex lumine cometæ debili perperam ejus a terra diſtantia colligi poteſt. Deinde ob inſignem illam radiorum lucis refractionem fieri poteſt, ut etiam maxima hemiſphærii a ſole averſi pars in cometa illuminetur; hincque nunquam in cometa phaſes, ſicuti in Luna & Venere obſervari ſolent. Atque hic noviſſimus cometa ſemper pleno lumine ſplendere eſt obſervatus, etiamſi in quadrato ſolis eſſet verſatus, quo tempore falcatus conſpici debuiſſet, niſi tanta refractio affuiſſet.

TABULA
MOTUS COMETAE IN PARABOLA.

Ad finem diſſertationis de cometa adjungenda.

v An. vera	$t + \frac{1}{3} t^3$ AreaParab.	$l(t + \frac{1}{3} t^3)$ Logar. Ar.	v An. vera	$t + \frac{1}{3} t^3$ AreaParab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.
grad. 0	0, 0000000	$-\infty$	grad. 9	0, 0788642	8, 8968799
	87271	∞		88477	461787
1.	0, 0087271	7, 9408700	10.	0, 0877119	8, 9430586
	87297	3010946		88747	418584
2.	0, 0174568	8, 2419646	11.	0, 0965866	8, 9849170
	87351	1752024		89046	382992
3.	0, 0261919	8, 4171670	12.	0, 1054912	9, 0232162
	87431	1260937		89374	353183
4.	0, 0349350	8, 5432607	13.	0, 1144286	9, 0585345
	87536	971075		89731	327867
5.	0, 0436886	8, 6403682	14.	0, 1234017	9, 0913212
	87672	794253		90114	306097
6.	0, 0524558	8, 7197935	15.	0, 1324131	9, 1219309
	87831	672339		90530	287215
7.	0, 0612389	8, 7870274	16.	0, 1414661	9, 1506524
	88019	583237		90976	270679
8.	0, 0700408	8, 8453511	17.	0, 1505637	9, 1777203
	88234	515288		91451	255974
9.	0, 0788642	8, 8968799	18.	0, 1597088	9, 2033177

Nachricht an den Buchbinder: dieſe Tabelle wird zwiſchen p. 90. und 91. gebunden.

v An.vera	$t+\frac{1}{3}t^3$ Ar. Parob.	$l(t+\frac{1}{3}t^3)$ Logar. Ar.	v An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo. Ar.Pas.
grad. 18	0, 1597088	9, 2033177	grad. 33	0, 3048770	9, 4841247
	91959	243240		103816	145422
19.	0, 1689047	9, 2276417	34.	0, 3152586	9, 4986669
	92497	231548		104885	142137
20.	0, 1781544	9, 2507965	35.	0, 3257471	9, 5128806
	93068	221149		106068	139059
21.	0, 1874612	9, 2729114	36.	0, 3363539	9, 5267965
	93672	211763		107278	136352
22.	0, 1968284	9, 2940877	37.	0, 3470817	9, 5404317
	94310	203259		108539	133731
23.	0, 2062594	9, 3144136	38.	0, 3579356	9, 5538048
	94982	195525		109852	131283
24.	0, 2157576	9, 3339661	39.	0, 3689208	9, 5669331
	95691	188466		111216	128990
25.	0, 2253267	9, 3528127	40.	0, 3800424	9, 5798321
	96433	181997		112641	126850
26.	0, 2349700	9, 3710124	41.	0, 3913065	9, 5925171
	97212	176060		114118	124843
27.	0, 2446912	9, 3886184	42.	0, 4027183	9, 6050014
	98033	170599		115660	122971
28.	0, 2544945	9, 4056783	43.	0, 4142843	9, 6172985
	98888	165557		117260	121216
29.	0, 2643833	9, 4222340	44.	0, 4260103	9, 6294201
	99784	160895		118926	119577
30.	0, 2743617	9, 4383235	45.	0, 4379029	9, 6413778
	100724	156582		120658	118045
31.	0, 2844341	9, 4539817	46.	0, 4499687	9, 6531823
	101703	152575		122459	116614
32.	0, 2946044	9, 4692392	47.	0, 4622146	9, 6648437
	102726	148855		124331	115276
33.	0, 3048770	9, 4841247	48.	0, 4746477	9, 6763713

v An. vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo. Ar. Par.	v An. vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo. Ar. Par.
grad. 48	0, 4746477	9, 6763713	grad. 62.	0, 6731708	9, 8281252
	126281	114036		163373	104142
49.	0, 4872758	9, 6877749	63.	0, 6895081	9, 8385394
	128303	112873		166905	103875
50.	0, 5001061	9, 6990622	64.	0, 7061986	9, 8489269
	130409	111796		170584	103657
51.	0, 5131470	9, 7102418	65.	0, 7232570	9, 8592926
	132601	110800		174422	103493
52.	0, 5264071	9, 7213218	66.	0, 7406992	9, 8696419
	134876	109873		178421	103374
53.	0, 5398947	9, 7323091	67.	0, 7585413	9, 8799793
	137244	109020		182590	103300
54.	0, 5536191	9, 7432111	68.	0, 7768003	9, 8903093
	139707	108235		186942	103279
55.	0, 5675898	9, 7540346	69.	0, 7954945	9, 9006372
	142270	107517		191480	103298
56.	0, 5818168	9, 7647863	70.	0, 8146425	9, 9109670
	144935	106860		196220	103368
57.	0, 5963103	9, 7754723	71.	0, 8342645	9, 9213038
	147708	106266		201166	103479
58.	0, 6110811	9, 7860989	72.	0, 8543811	9, 9316517
	150594	105729		206334	103635
59.	0, 6261405	9, 7966718	73.	0, 8750145	9, 9420152
	153598	105251		211731	103837
60.	0, 6415003	9, 8071969	74.	0, 8961876	9, 9523989
	156725	104827		217378	104084
61.	0, 6571728	9, 8176796	75.	0, 9179254	9, 9628073
	159980	104456		223277	104375
62.	0, 6731708	9, 8281252	76.	0, 9402531	9, 9732448

v An. ver.	$t + \frac{1}{3} t^3$ Ar. Parab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.	v An. ver.	$t + \frac{1}{3} t^3$. Ar. Parab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.
76.	0, 9402531	9, 9732448	91.	1, 3688601	10,1363592
	229451	104709		368111	115245
77.	0, 9631982	9, 9837157	92.	1, 4056712	10, 1478837
	235911	105087		381665	116346
78.	0, 9867893	9, 9942244	93.	1, 4438377	10, 1595183
	242677	105512		395965	117500
79.	1, 0110570	10, 0047756	94.	1, 4834342	10, 1712683
	249760	105982		411070	118708
80.	1, 0360330	10, 0153738	95.	1, 5245412	10.1831391
	257185	106491		427036	119978
81.	1, 0617515	10,0260229	96.	1, 5672448	10,1951369
	264969	107052		443914	121302
82.	1, 0882484	10,0367281	97.	1, 6116362	10,2072671
	273131	107655		461777	122686
83.	1, 1155615	10,0474936	98.	1, 6578139	10,2195357
	281699	108304		480706	124140
84.	1, 1437314	10,0583240	99.	1, 7058845	10,2319497
	290689	109001		500758	125650
85.	1, 1728003	10,0692241	100.	1, 7559603	10,2445147
	300141	109749		522027	127129
86.	1, 2028154	10, 0801990	101.	1, 8081630	10,2572376
	310074	110539		544617	128869
87.	1, 2338228	10, 0912529	102.	1, 8626247	10,2701245
	320519	111378		568605	130604
88.	1, 2658747	10, 1023907	103.	1, 9194852	10,2831849
	331508	112270		594116	132373
89.	1, 2990255	10, 1136177	104.	1, 9788968	10,2964222
	343078	113209		621261	134256
90.	1, 3333333	10, 1249386	105.	2, 0410229	10,3098478
	355268	114206		650185	136192
91.	1, 3688601	10, 1363592	106.	2, 1060414	10,3234670

v An. ver.	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.	v An. ver.	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.
106.	2, 1060414	10,3234670	120.	3, 4641016	10,5395906
	681009	138210		1439648	176840
107.	2, 1741423	10,3372880	121.	3, 6080664	10,5572746
	715916	140702		1531254	180508
108.	2, 2457339	10,3513582	122.	3, 7611918	10,5753254
	747051	142120		1630816	184339
109.	2, 3204390	10,3655702	123.	3, 9242734	10,5937593
	786624	144783		1739121	188323
110.	2, 3991014	10,3800485	124.	4, 0981855	10,6125916
	826836	147157		1857152	192478
111.	2, 4817850	10,3947642	125.	4, 2839007	10,6318394
	869928	149623		1985965	196806
112.	2, 5687778	10,4097265	126.	4, 4824972	10,6515200
	916132	152190		2126822	201322
113.	2, 6603910	10,4249455	127.	4, 6951794	10,6716522
	965746	154857		2281094	206031
114.	2, 7569656	10,4404312	128.	4, 9232888	10,6922553
	1019103	157641		2450375	210947
115.	2, 8588759	10,4561953	129.	5, 1683263	10,7133500
	1076486	160525		2636466	216076
116.	2, 9665245	10,4722478	130.	5, 4319729	10,7349576
	1138350	163535		2841418	221434
117.	3, 0803595	10,4886013	131.	5, 7161147	10,7571010
	1205088	166665		3067691	227035
118.	3, 2008683	10,5052678	132.	6, 0228838	10,7798045
	1277168	169919		3317927	232890
119.	3, 3285851	10,5222597	133.	6, 3546765	10,8030935
	1355165	173309		3593349	238885
120.	3, 4641016	10,5395906	134.	6, 7140114	10,8269820

v An. ver.	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.	v An. ver.	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.
134.	6, 7140114	10,8269820	148.	17, 625464	11, 2461405
	3905592	245558		1, 608797	379350
135.	7, 1045706	10,8515378	149.	19, 234261	11, 2840755
	4246883	252144		1, 824706	393616
136.	7, 5292589	10,8767522	150.	21, 058967	11, 3234371
	4630260	259188		2, 078751	408835
137.	7, 9922849	10,9026710	151.	23, 137718	11, 3643206
	5059412	266572		2, 379354	425102
138.	8, 4982261	10,9293282	152.	25, 517072	11, 4068308
	5541169	274327		2, 737148	442525
139.	9, 0523430	10,9567609	153.	28, 254220	11, 4510833
	6083643	282480		3, 165844	461238
140.	9, 6607073	10,9850089	154.	31, 420064	11, 4972071
	669622	291052		3, 683004	481379
141.	10, 330329	11, 0141141	155.	35, 103068	11, 5453450
	739010	300076		4, 311613	503130
142.	11, 069339	11, 0441217	156.	39, 414681	11, 5956580
	817891	309590		5, 081856	526683
143.	11, 887230	11,0750807	157.	44, 496537	11, 6483263
	907868	319629		6, 034015	552278
144.	12, 795098	11, 1070436	158.	50, 530552	11, 7035541
	1, 010869	330233		7, 222367	580197
145.	13, 805967	11, 1400669	159.	57, 752919	11, 7615738
	1, 129269	341451		8, 720981	610774
146.	14, 935236	11, 1742120	160.	66, 473900	11, 8226512
	1, 265913	353338		10, 632908	644416
147.	16, 201149	11, 2095458	161.	77, 106808	11, 8870928
	1, 424315	365947		13, 102930	681607
148.	17, 625464	11, 2461405	162.	90, 209738	11, 9552535

v An. ver.	$t + \frac{1}{3} t^3$ Ar. Parab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.	v An. ver.	$t + \frac{1}{3} t^3$ Ar. Parab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.
162.	90, 209738	11, 9552535	171.	696, 50162	12,8429220
	16, 339289	722960		292, 67111	1523501
163.	106,549027	12,0275495	172.	989, 17273	12,9952721
	20, 64650	769223		484, 0461	1729950
164.	127, 19553	12,1044718	173.	1473, 2188	13,1682671
	26, 48042	821340		861, 6103	1999880
165.	153, 67595	12,1866058	174.	2334, 8291	13,3682551
	34, 54088	880526		1693, 0339	2368196
166.	188, 21683	12,2746584	175.	4027, 8630	13,6050747
	45, 93237	948342		3802, 5759	2887114
167.	234, 14920	12,3694926	176.	7830, 4389	13,8937861
	62, 45521	1026849		10737, 522	3749781
168.	296, 60441	12,4721775	177.	18567, 960	14,2687642
	87, 15848	1118855		44167, 208	5287469
169.	383, 76289	12,5840630	178.	62735, 168	14,7975111
	125, 43102	1228203		438917, 01	9028915
170.	509, 19391	12,7068833	179.	501652, 17	15,7004026
	187, 30771	1360387		∽	∽
171.	696, 50162	12,8429220	180.	infinitum.	infinitum.

Tabula I.

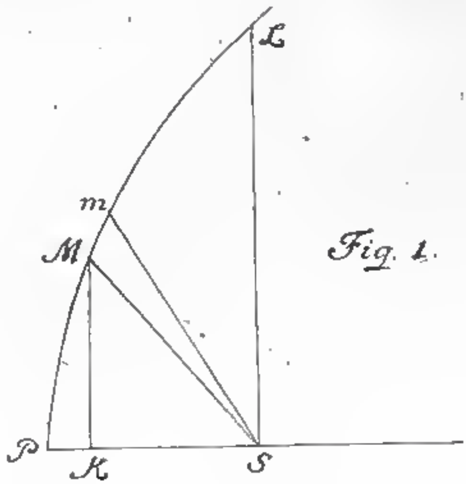


Fig. 1.

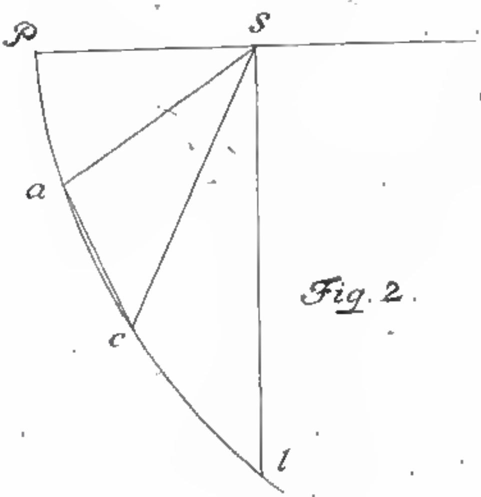


Fig. 2.

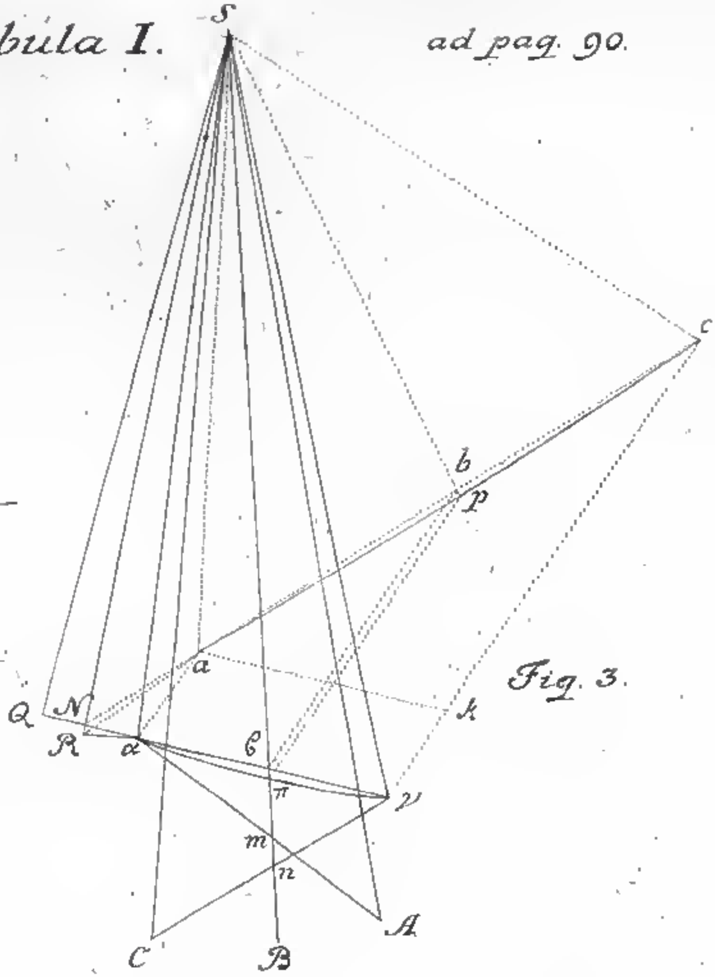


Fig. 3.

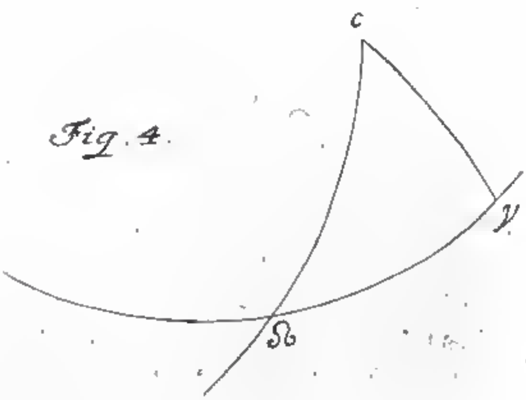


Fig. 4.

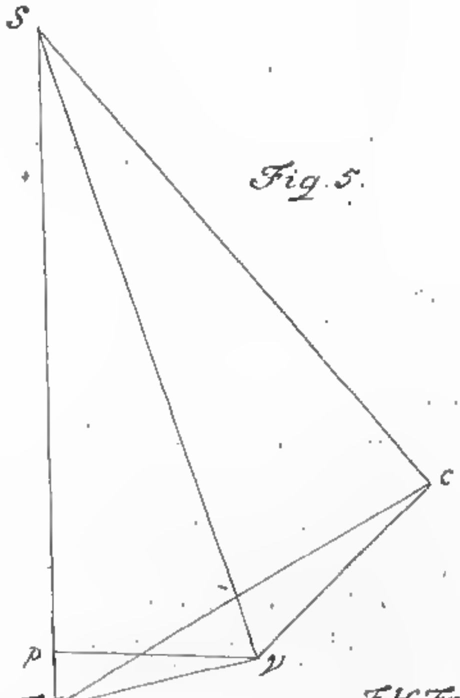


Fig. 5.

F.H.Frisch sc.